

# Relativistic SZ maps and electron temperature spectroscopy

Mathieu Remazeilles

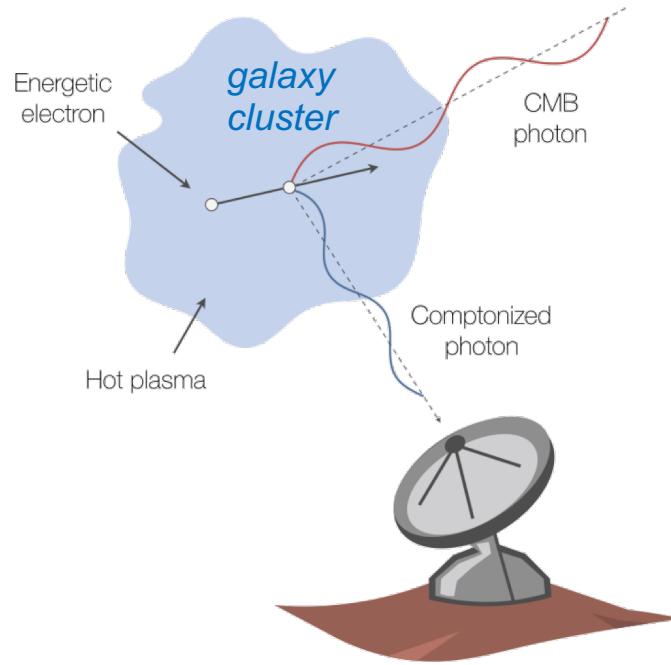


The University of Manchester

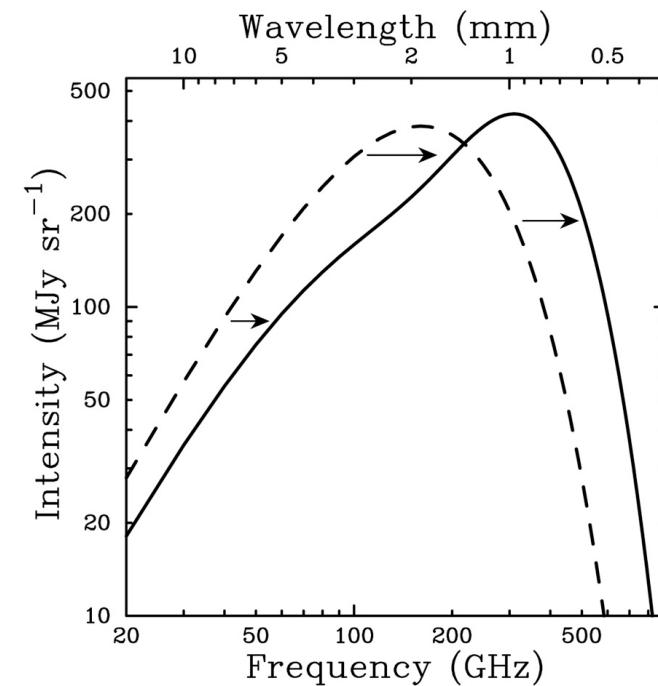
*Remazeilles & Chluba, MNRAS (2020)*  
*Remazeilles, Bolliet, Rotti, Chluba, MNRAS (2019)*

# Thermal Sunyaev-Zeldovich (SZ) Effect

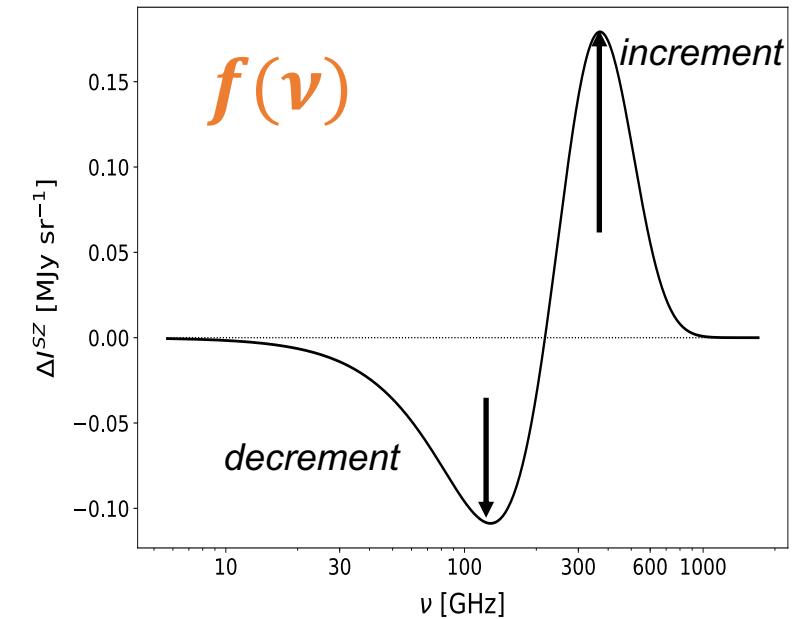
Zeldovich & Sunyaev 1969



*Inverse Compton scattering  
of CMB photons by hot gas  
of electrons in galaxy clusters*



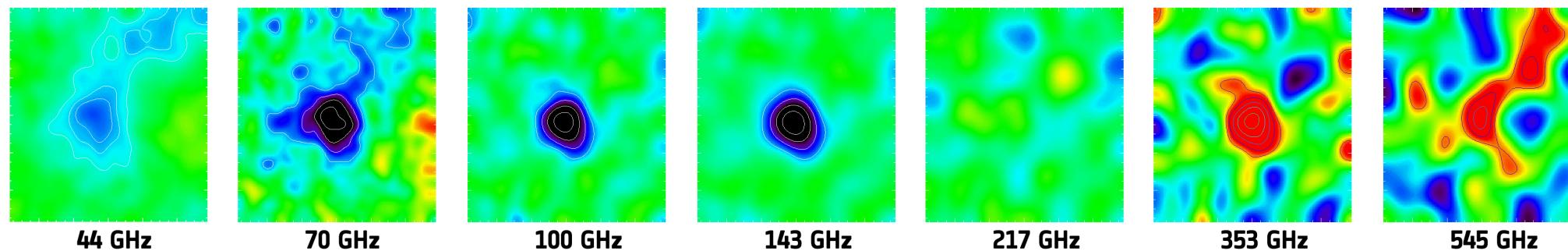
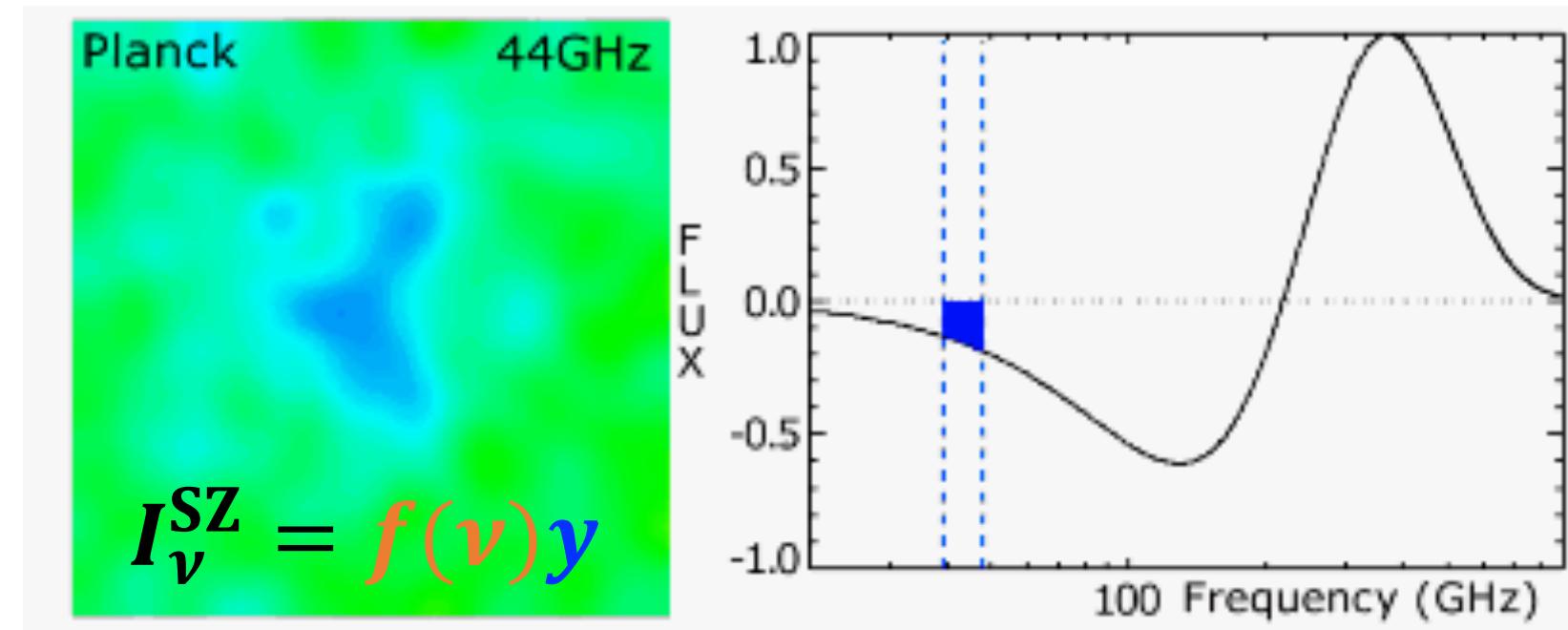
*y-type spectral distortion of  
CMB blackbody radiation*



*Spectral signature*

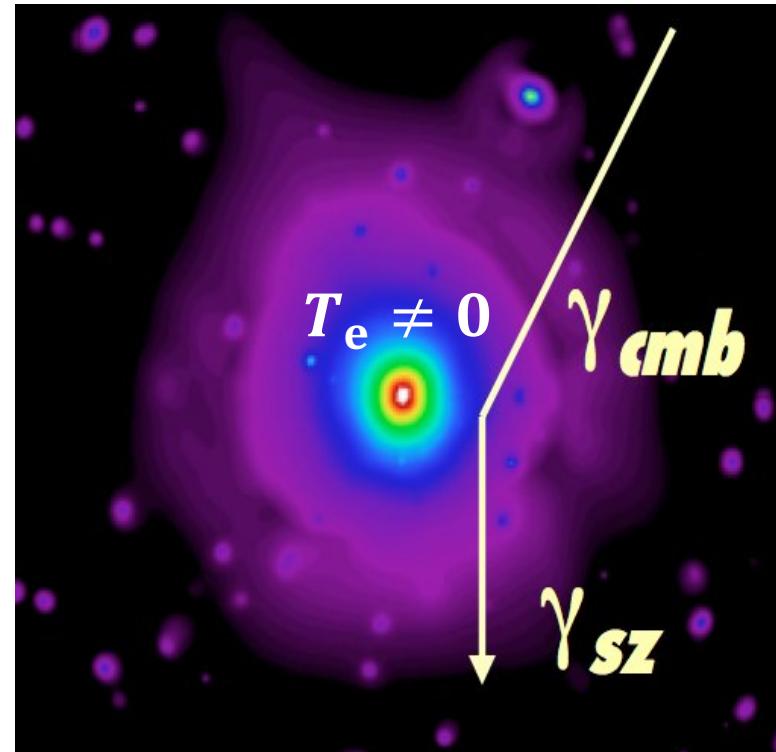
$$I_\nu^{\text{SZ}} \equiv \frac{\Delta I_\nu^{\text{CMB}}}{I_\nu^{\text{CMB}}} = f(\nu) \frac{\sigma_T}{m_e c^2} \int P_e(l) dl = f(\nu) y$$

# Spectroscopy of clusters across frequencies



*Credit: ESA/Planck Collaboration*

# Relativistic SZ effect (rSZ): Temperature corrections to thermal SZ effect



- Galaxy clusters are massive, so they are hot

Arnaud et al,  
A&A 2005

$$kT_e \simeq 5 \text{ keV} \left[ \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \frac{M_{500}}{3 \times 10^{14} h^{-1} M_\odot} \right]^{2/3}$$

- Thermal velocities of electrons approach the speed of light

$$v_e^{\text{th}} = \sqrt{2kT_e/m_e} \gtrsim 0.1c$$

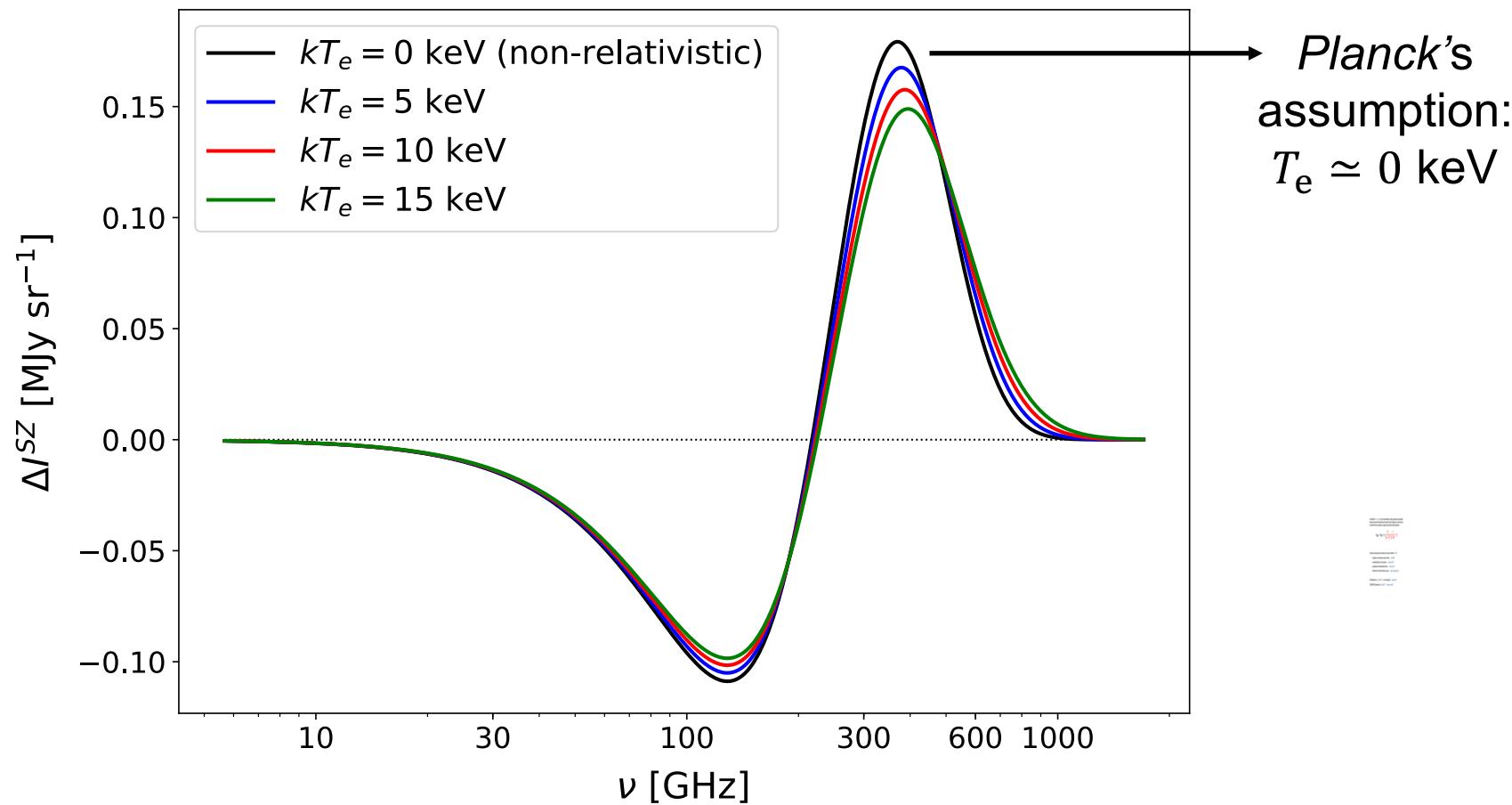
- Relativistic temperature corrections to the thermal SZ effect should be accounted for

$$I^{\text{SZ}}(\nu, \vec{n}) = f(\nu, T_e(\vec{n})) y(\vec{n})$$

*The spectral signature of SZ emission from galaxy clusters changes with the local electron gas temperature*

# Relativistic SZ temperature corrections

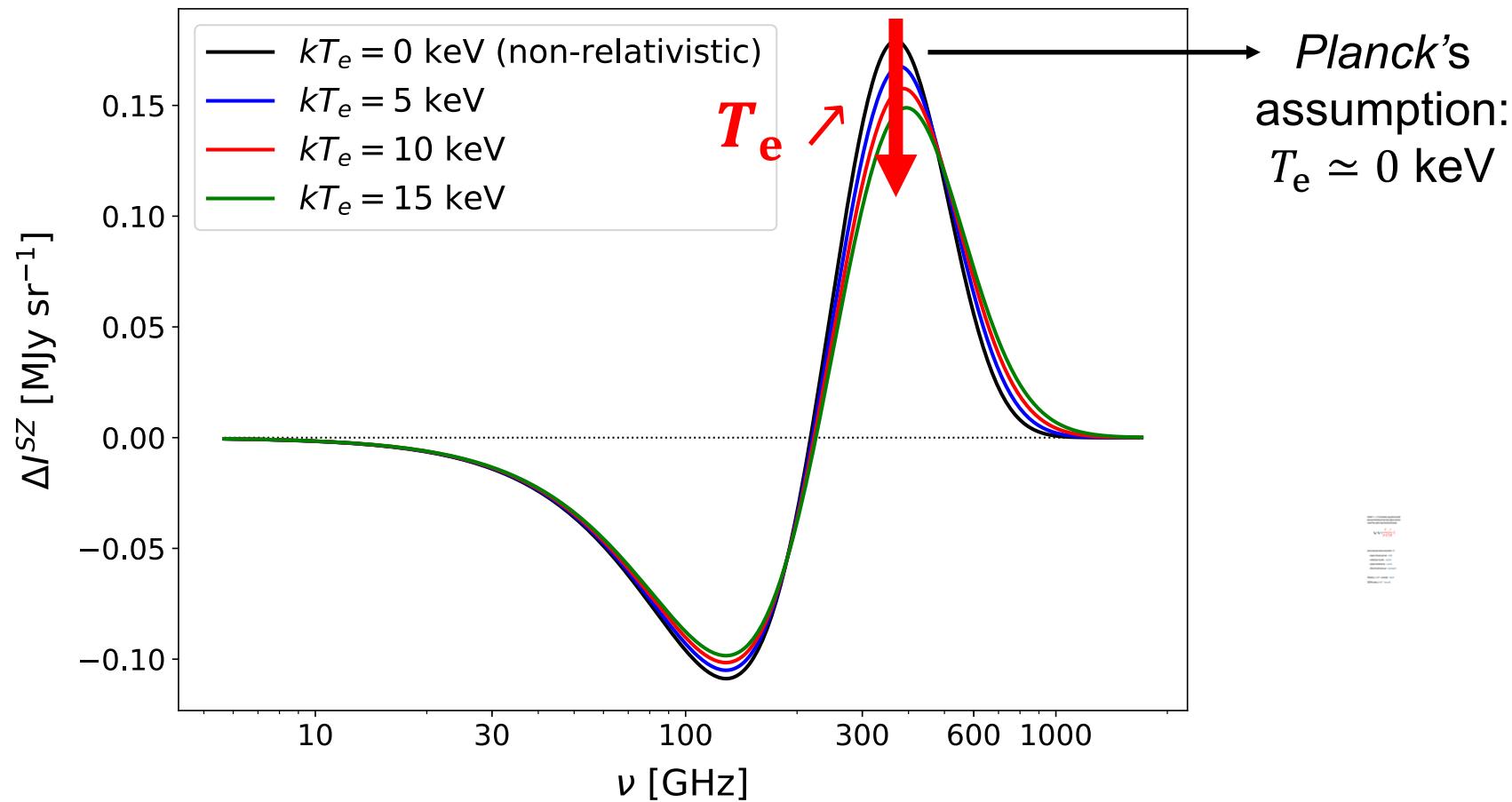
$$I_{\nu}^{\text{SZ}} = f(\nu, T_e(\vec{n})) y(\vec{n})$$



*The spectral signature of SZ emission from galaxy clusters changes with the local electron gas temperature*

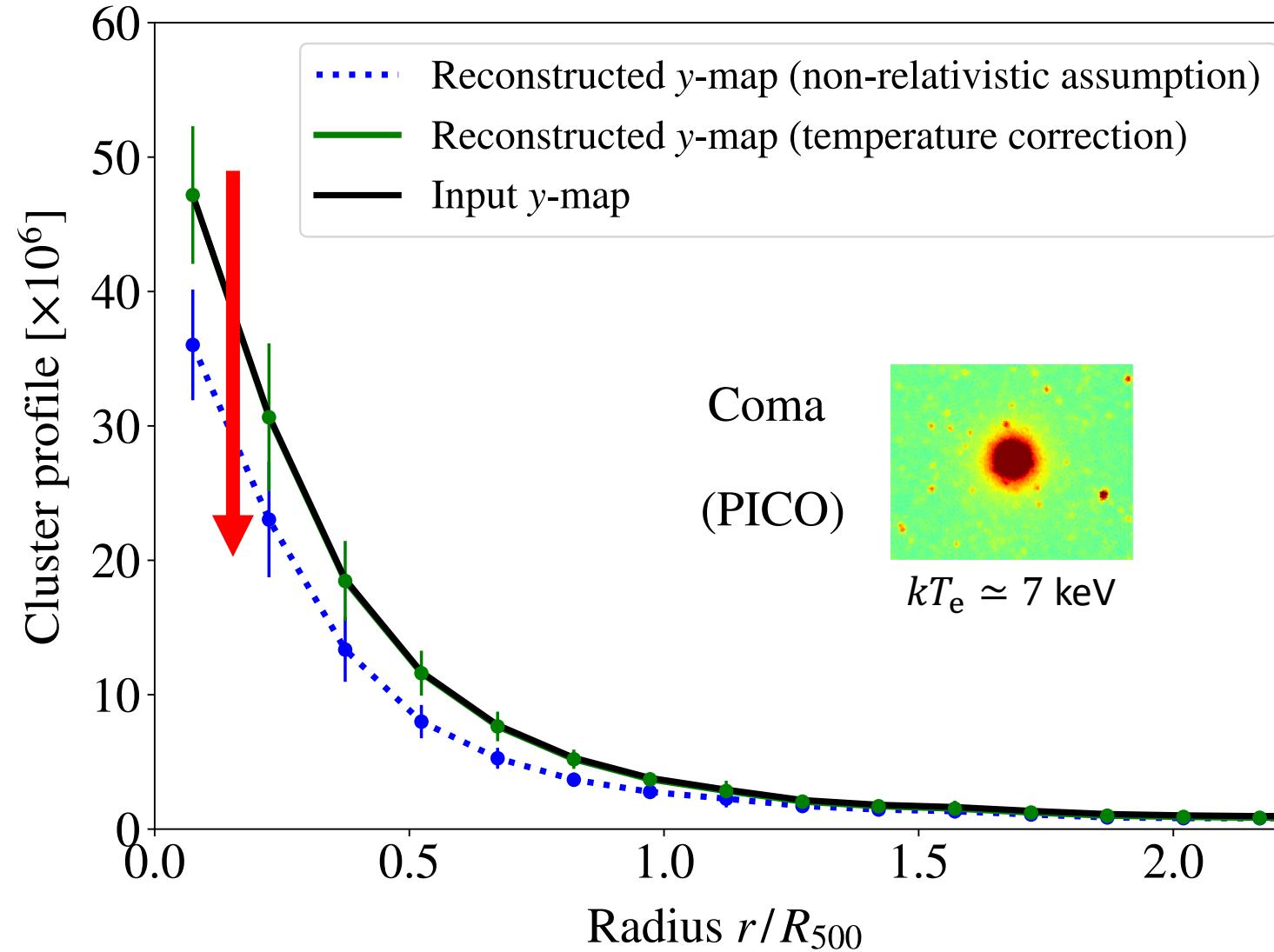
# Relativistic SZ temperature corrections

$$I_{\nu}^{\text{SZ}} = f(\nu, T_e(\vec{n})) y(\vec{n})$$

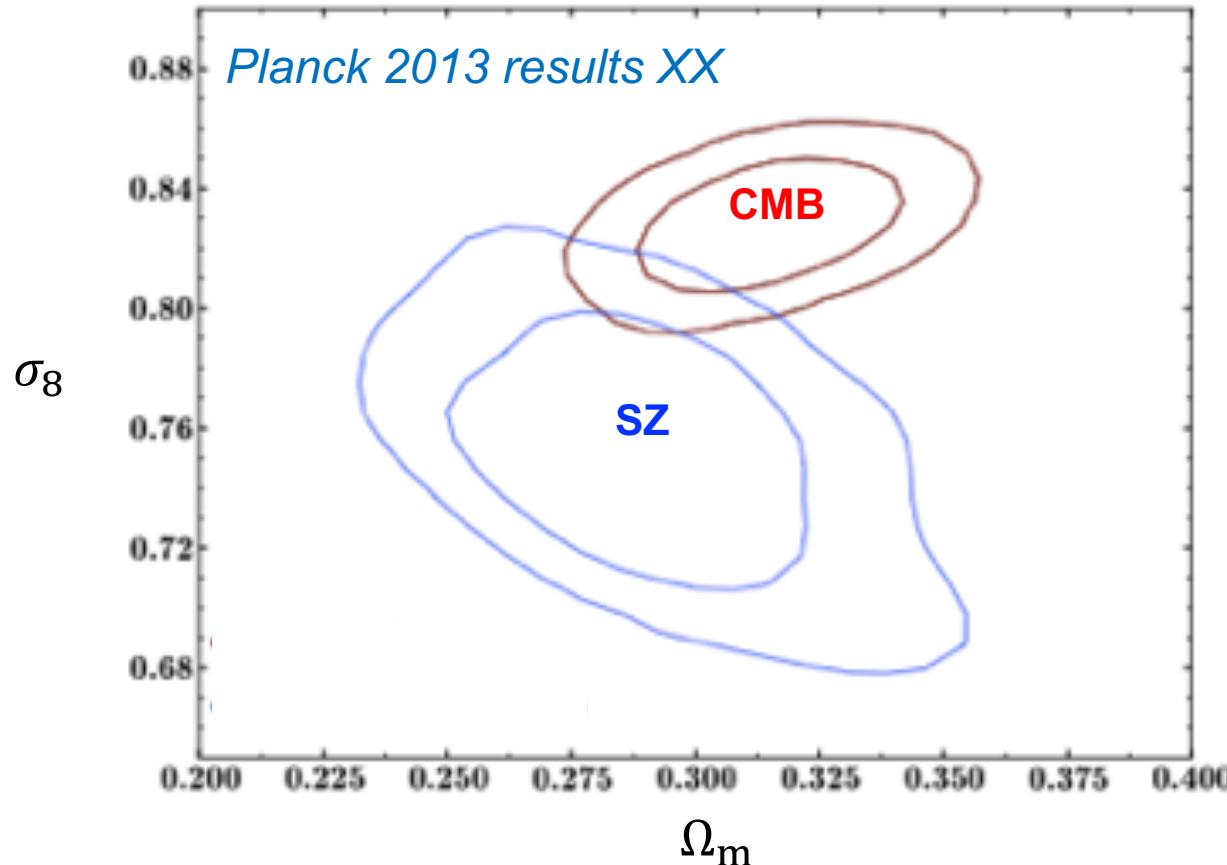


- Relativistic temperature corrections reduce the overall SZ intensity at fixed Compton- $y$  parameter
- Assuming the non-relativistic SED  $f(\nu, T_e = 0)$  underestimates the Compton- $y$  parameter

# Impact on cluster pressure profiles of neglecting relativistic SZ corrections



# Planck tension on $\sigma_8$ between CMB and SZ

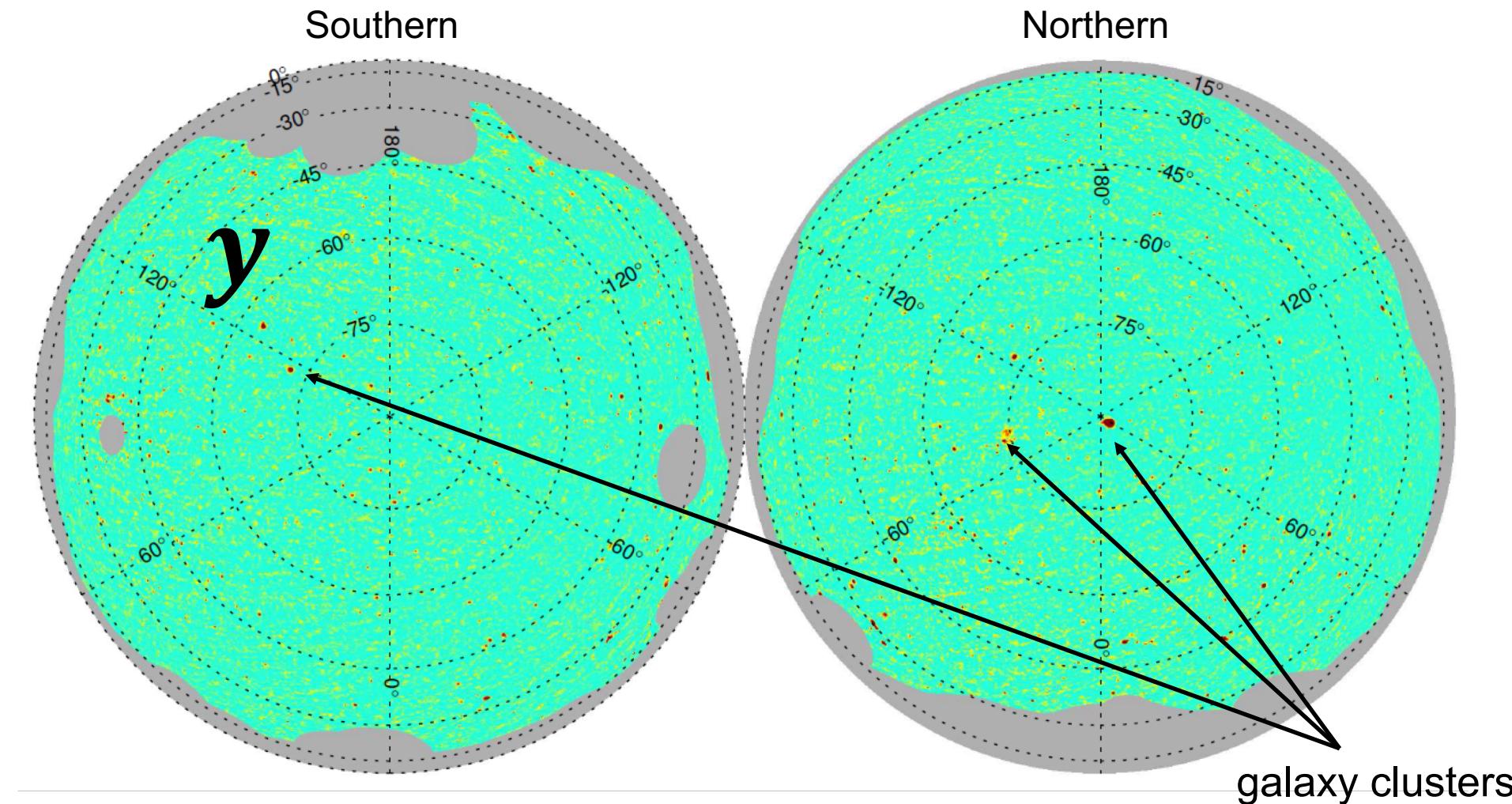


- Incompleteness of  $\Lambda$ CDM model?  
*Evidence for massive neutrinos?*
- Incorrect mass-bias in the Y-M relation?  
*Hydrodynamical simulations predict*  
 $M_{\text{SZ}}/M_{\text{dark matter}} = (1 - b) \approx 0.8$
- Miscalibrated Planck SZ analysis because of neglecting relativistic corrections?

*Remazeilles, Bollaert, Rotti, Chluba, MNRAS (2019)*

# The *Planck* SZ Compton $y$ -map

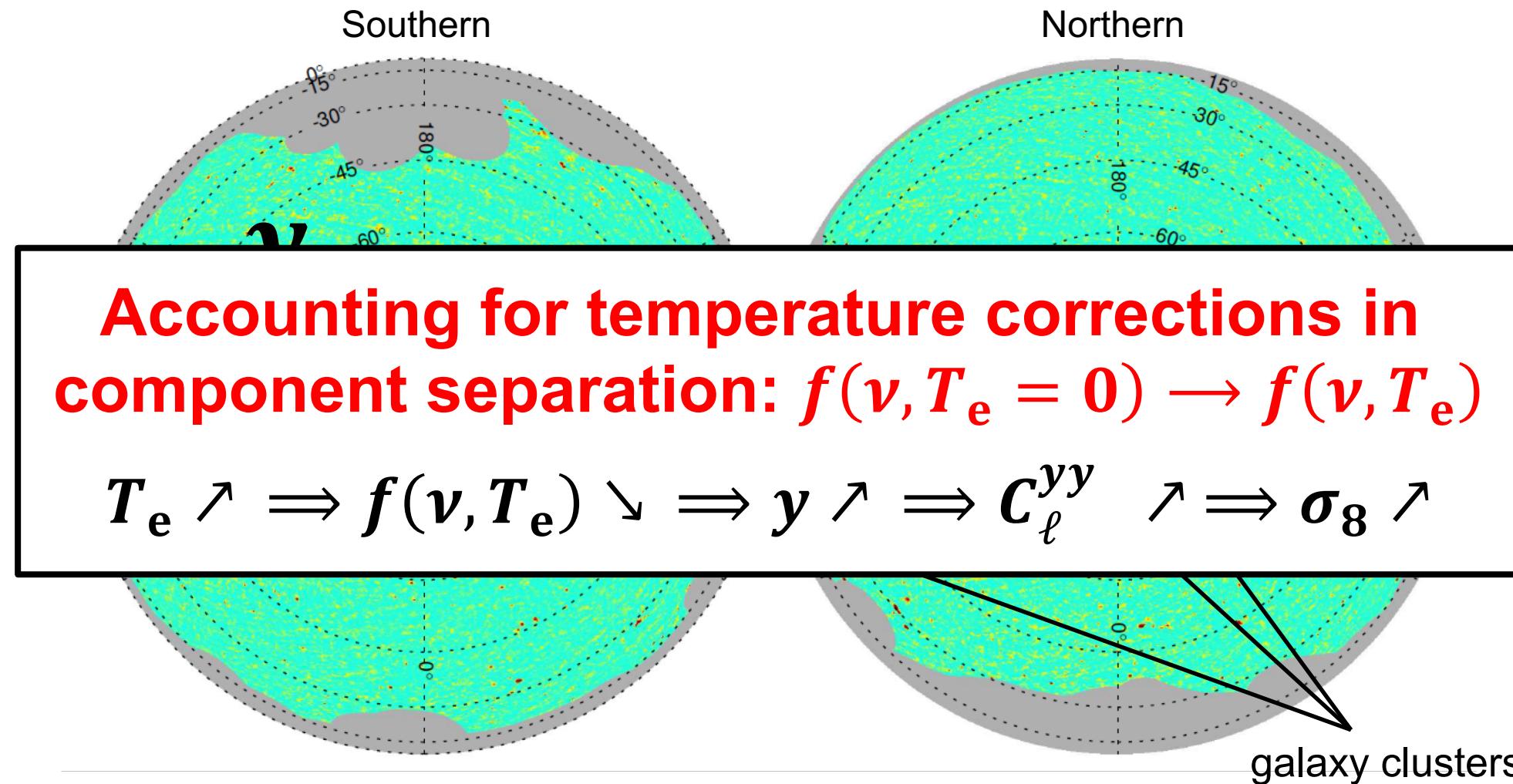
Non-relativistic approximation  $f(\nu, T_e = 0)$  for component separation



*Planck 2015 results XXII, A&A (2016)*

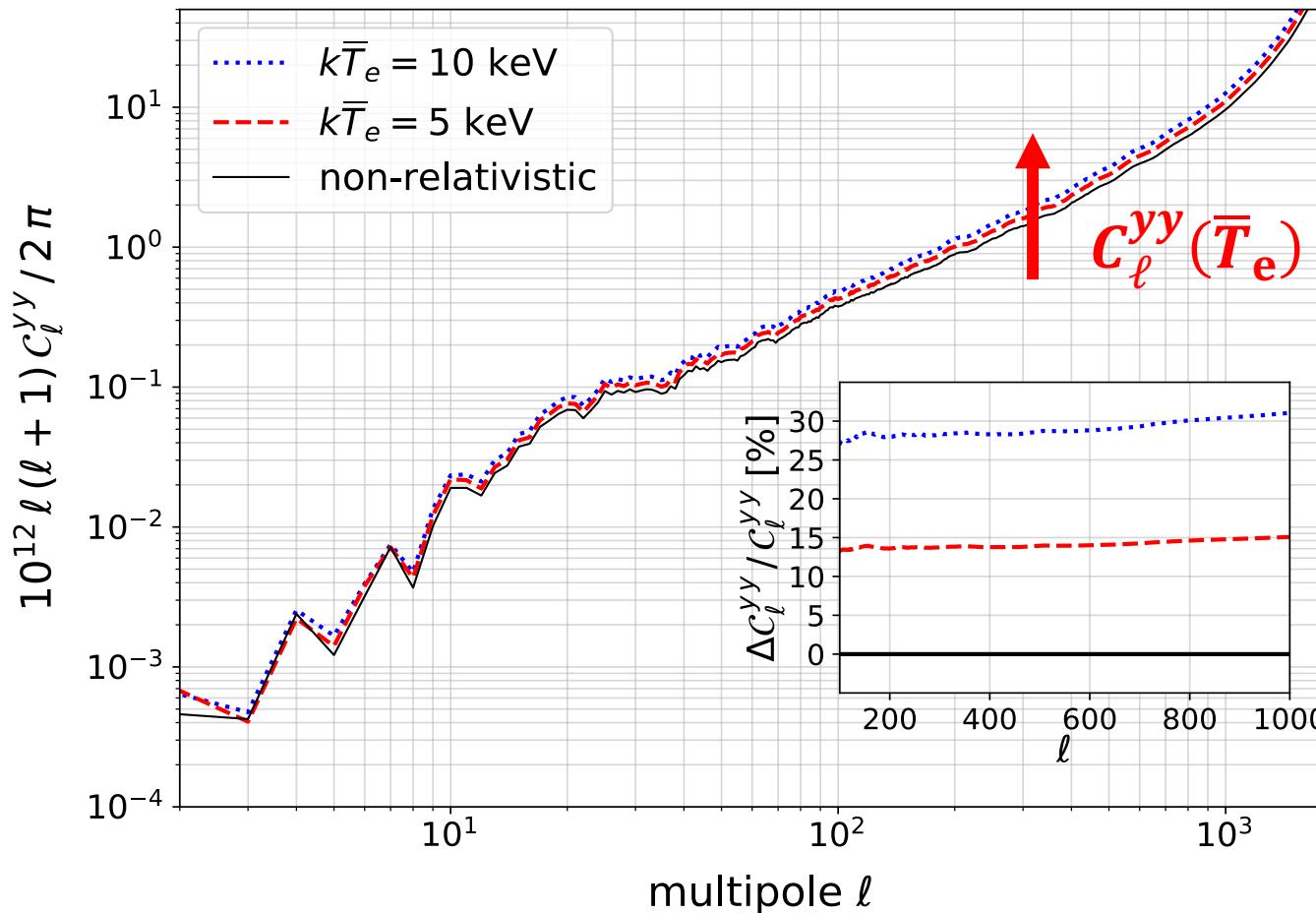
# Revisiting the *Planck* SZ Compton $y$ -map

*Remazeilles, Bolliet, Rotti, Chluba, MNRAS (2019)*



# Relativistic temperature corrections to the Planck SZ power spectrum

Remazeilles, Bollaert, Rotti, Chluba, MNRAS 2019



$c_\ell^{\gamma\gamma}(\bar{T}_e)$  increases with the average temperature  $\bar{T}_e$

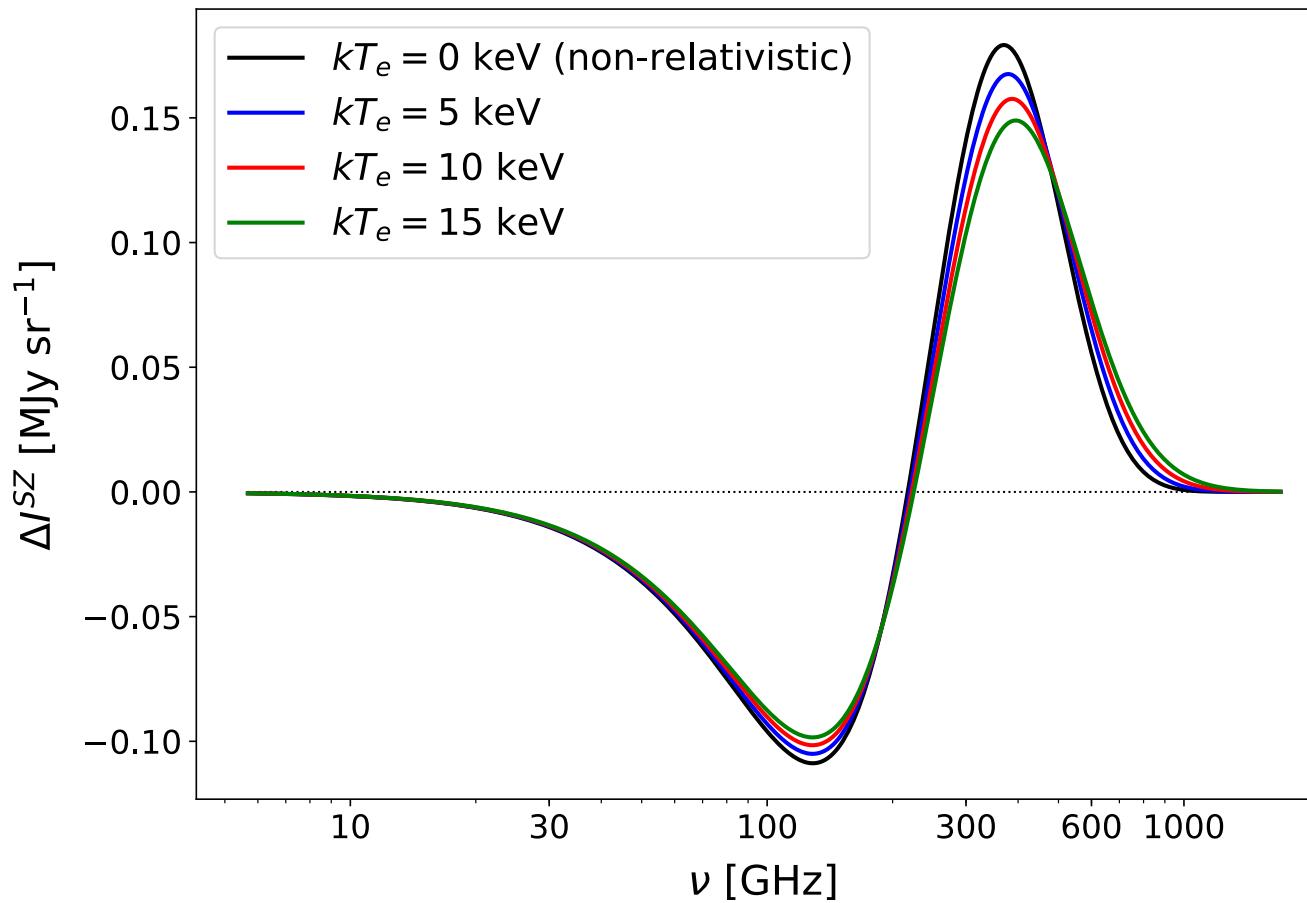
$$c_\ell^{\gamma\gamma} \sim \sigma_8^{8.1} \Rightarrow \frac{\Delta\sigma_8}{\sigma_8} \simeq 0.019 \left( \frac{k\bar{T}_e}{5 \text{ keV}} \right)$$

$k\bar{T}_e \simeq 5 \text{ keV}$  alleviates  
Planck's tension by  $1\sigma$

Mapping relativistic electron temperatures?

# Relativistic SZ temperature corrections

$$I_{\nu}^{\text{SZ}} = f(\nu, \mathbf{T}_e(\vec{n})) y(\vec{n})$$

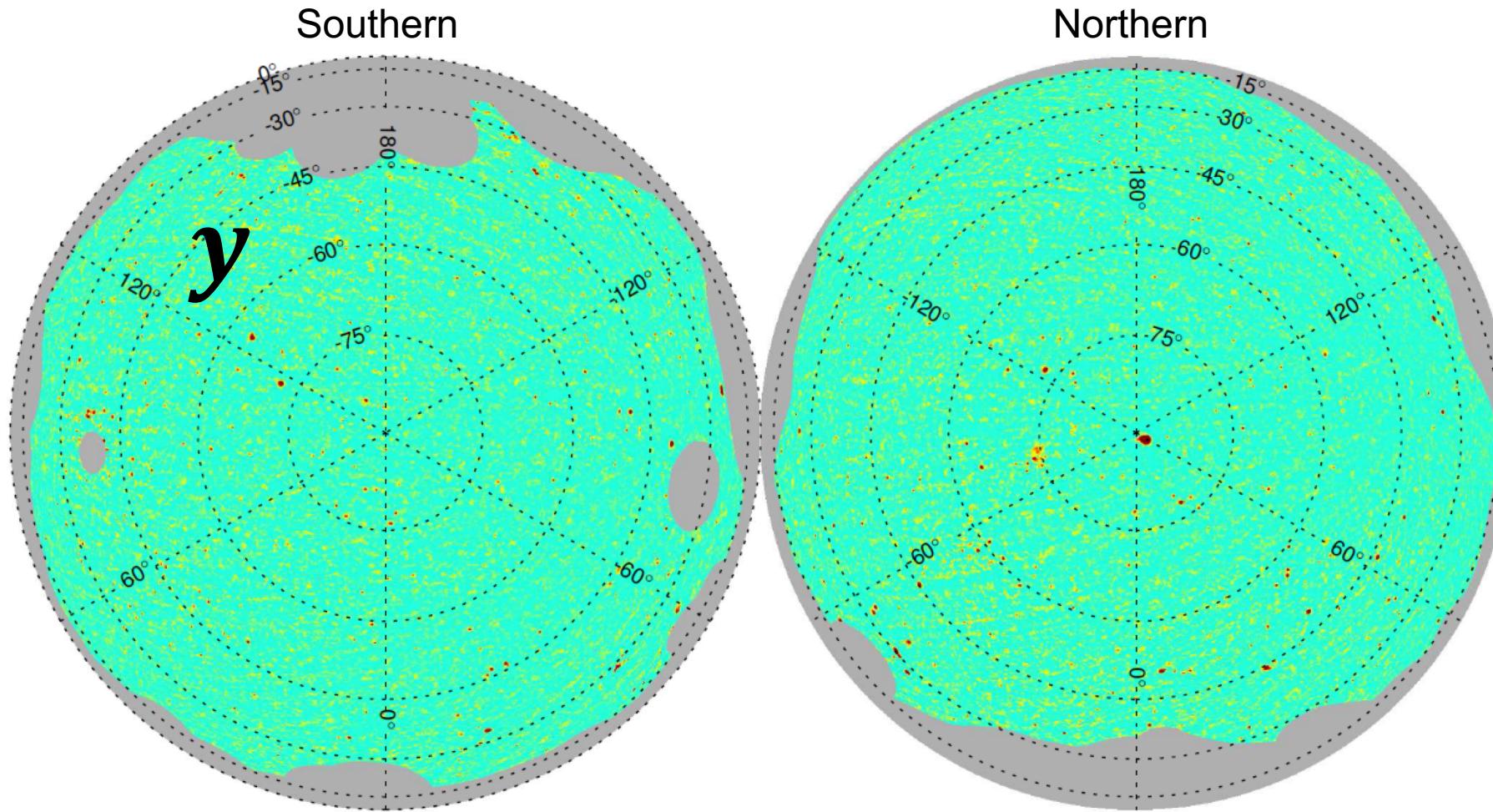


*Relativistic electron temperatures  
distort the shape of the SZ spectrum*

*Two complementary observables*

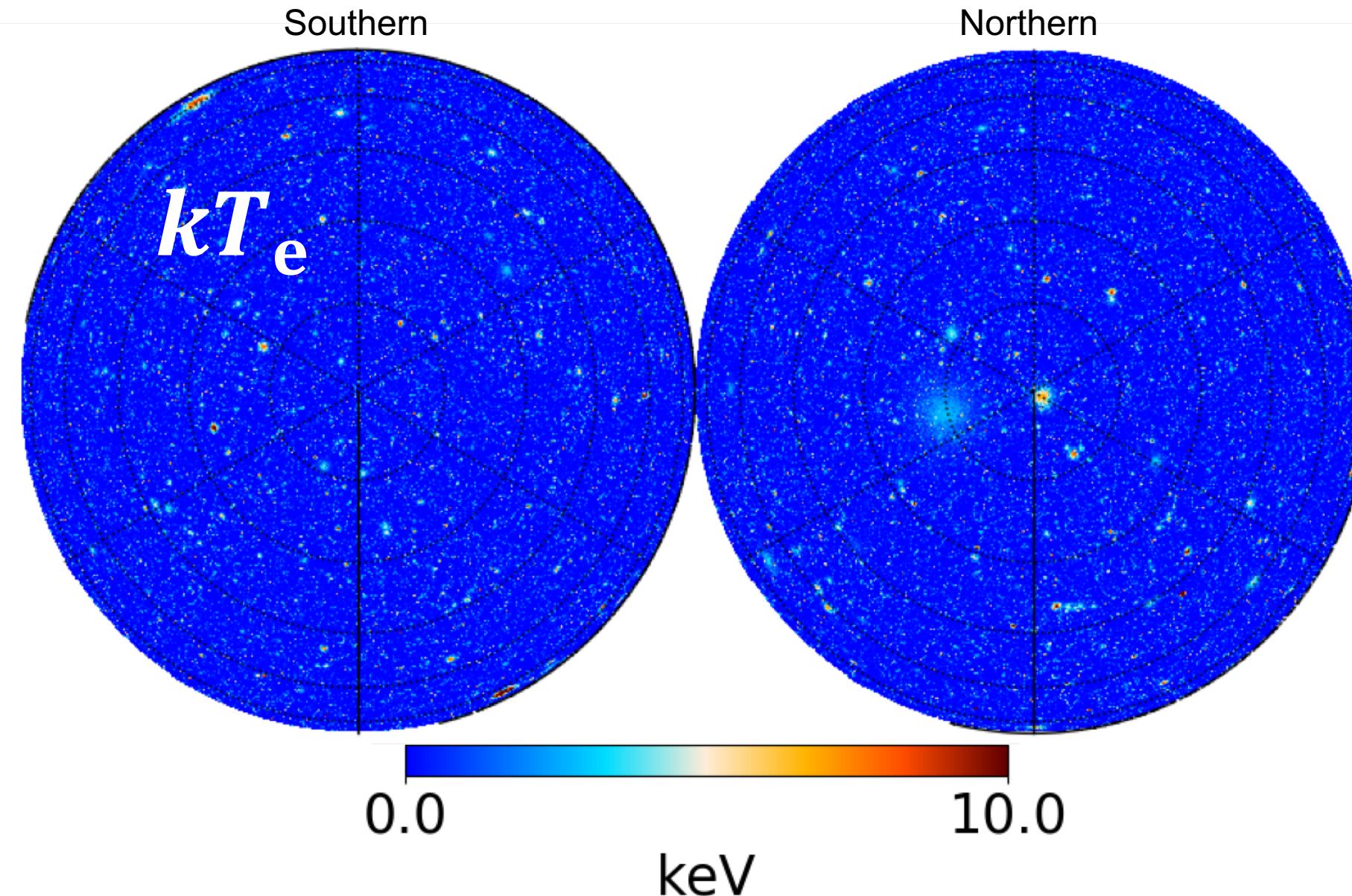
$$y(\vec{n}), \mathbf{T}_e(\vec{n})$$

# “First SZ revolution”: The *Planck* Compton $y$ -map



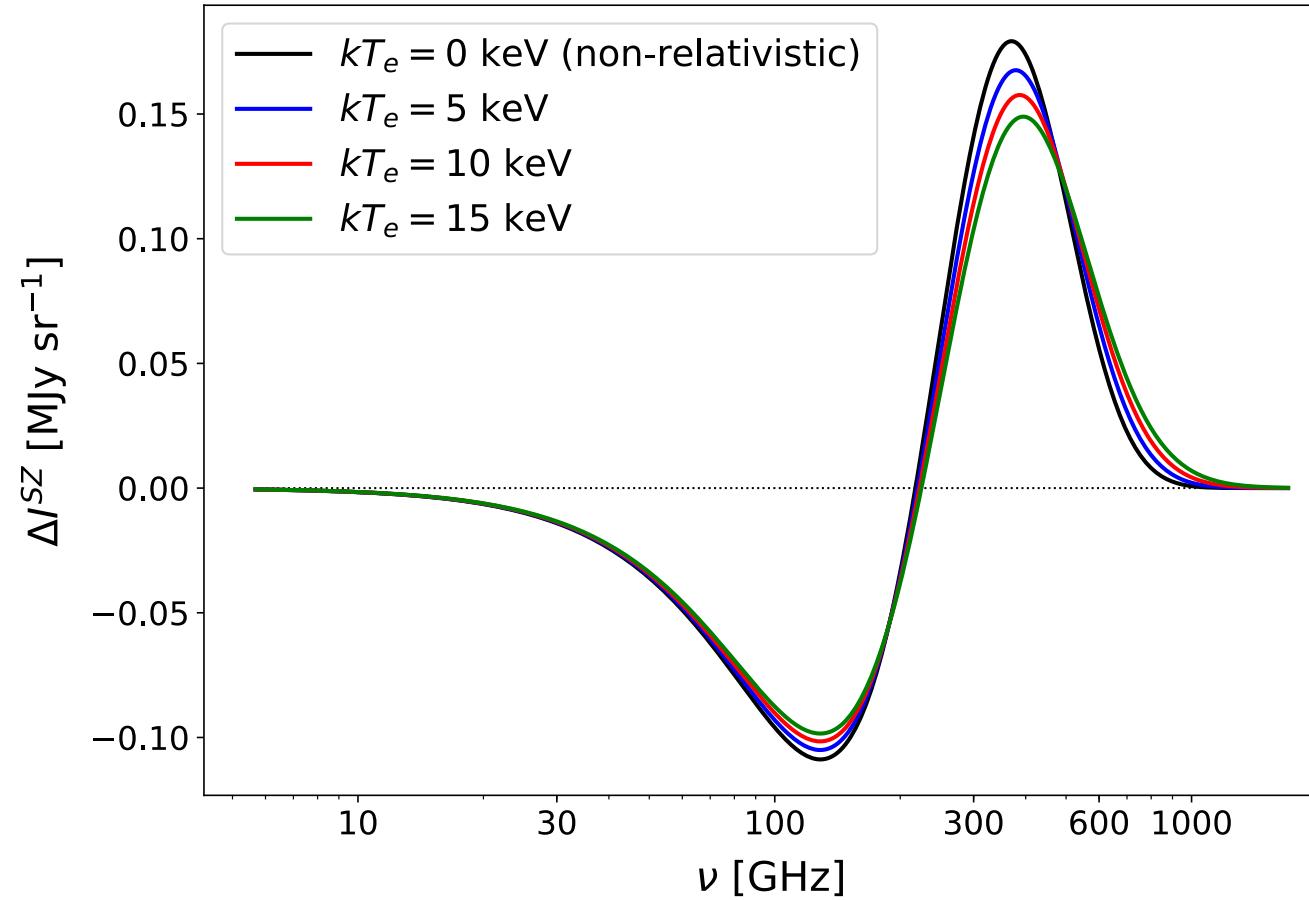
*Planck 2015 results XXII, A&A (2016)*

# “Second SZ revolution”: The electron temperature $T_e$ -map ?



# Relativistic SZ temperature corrections

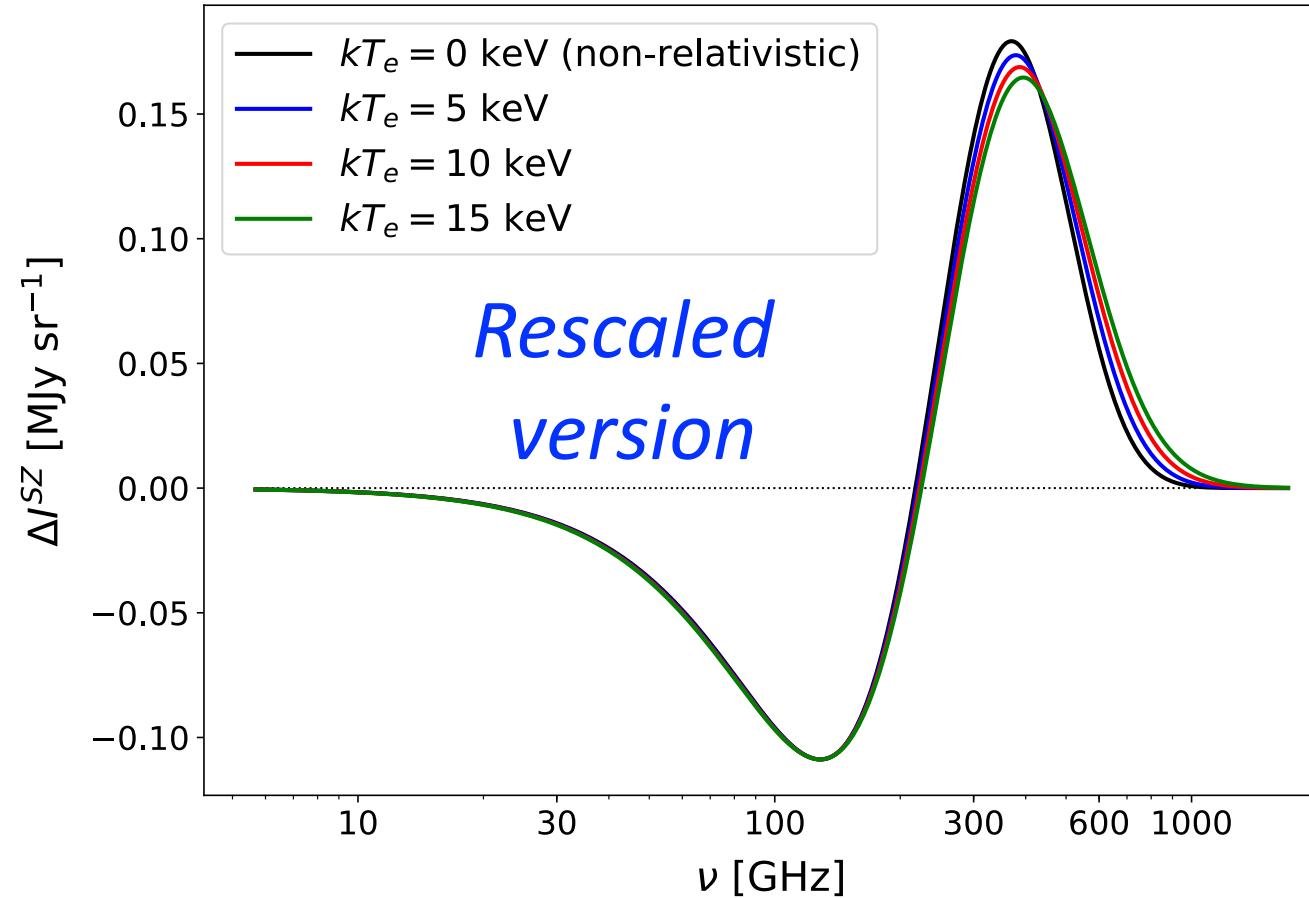
$$I_{\nu}^{\text{SZ}} = f(\nu, \textcolor{red}{T_e(\vec{n})}) y(\vec{n})$$



*The spectral signature of SZ emission from galaxy clusters changes with the local electron gas temperature*

# Relativistic SZ temperature corrections

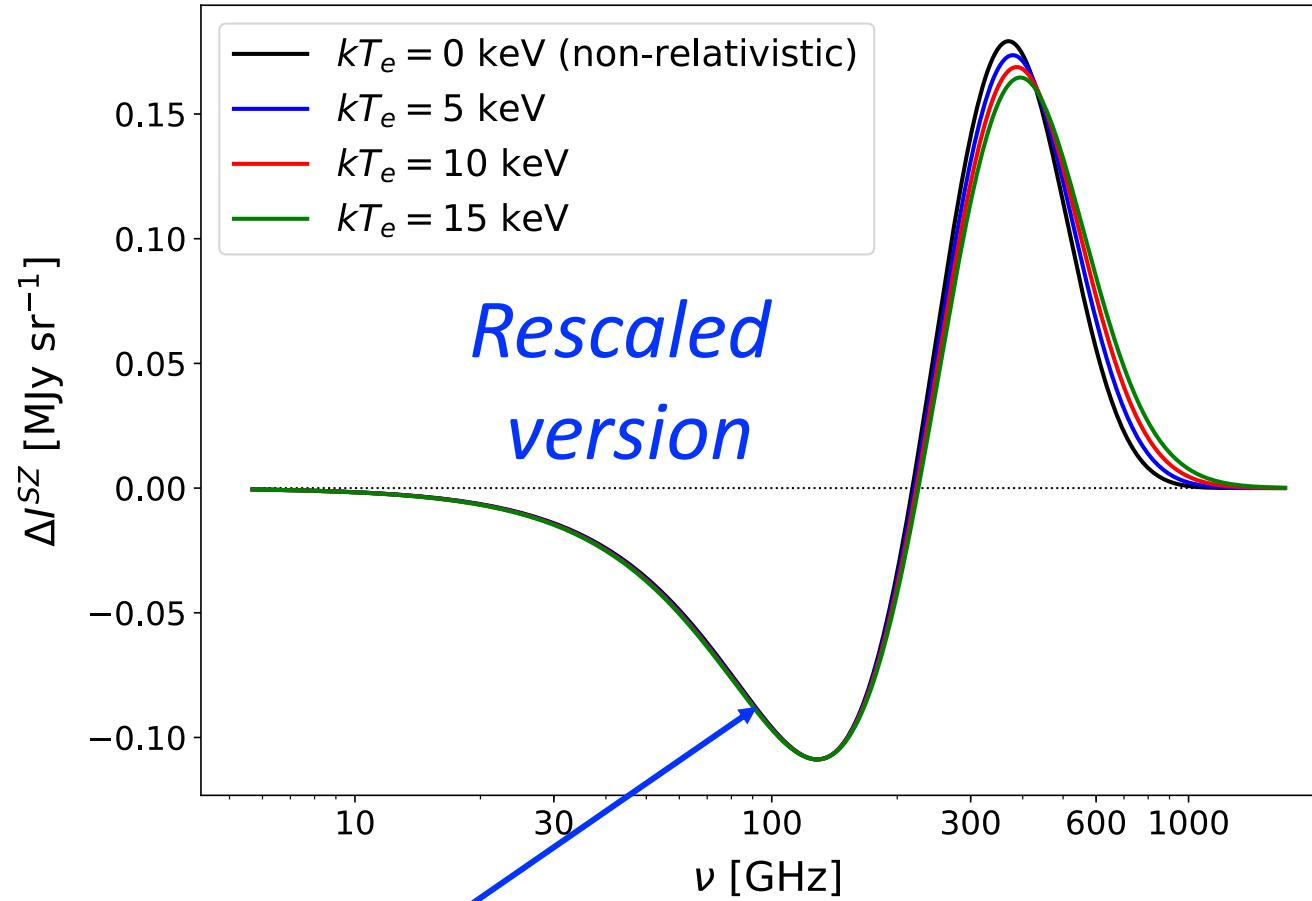
$$I_{\nu}^{\text{SZ}} = f(\nu, T_e(\vec{n})) y(\vec{n})$$



*The spectral signature of SZ emission from galaxy clusters  
changes with the local electron gas temperature*

# The $y$ - $T_e$ degeneracy at low frequency

$$I_\nu^{\text{SZ}} = f(\nu, T_e(\vec{n})) y(\vec{n})$$

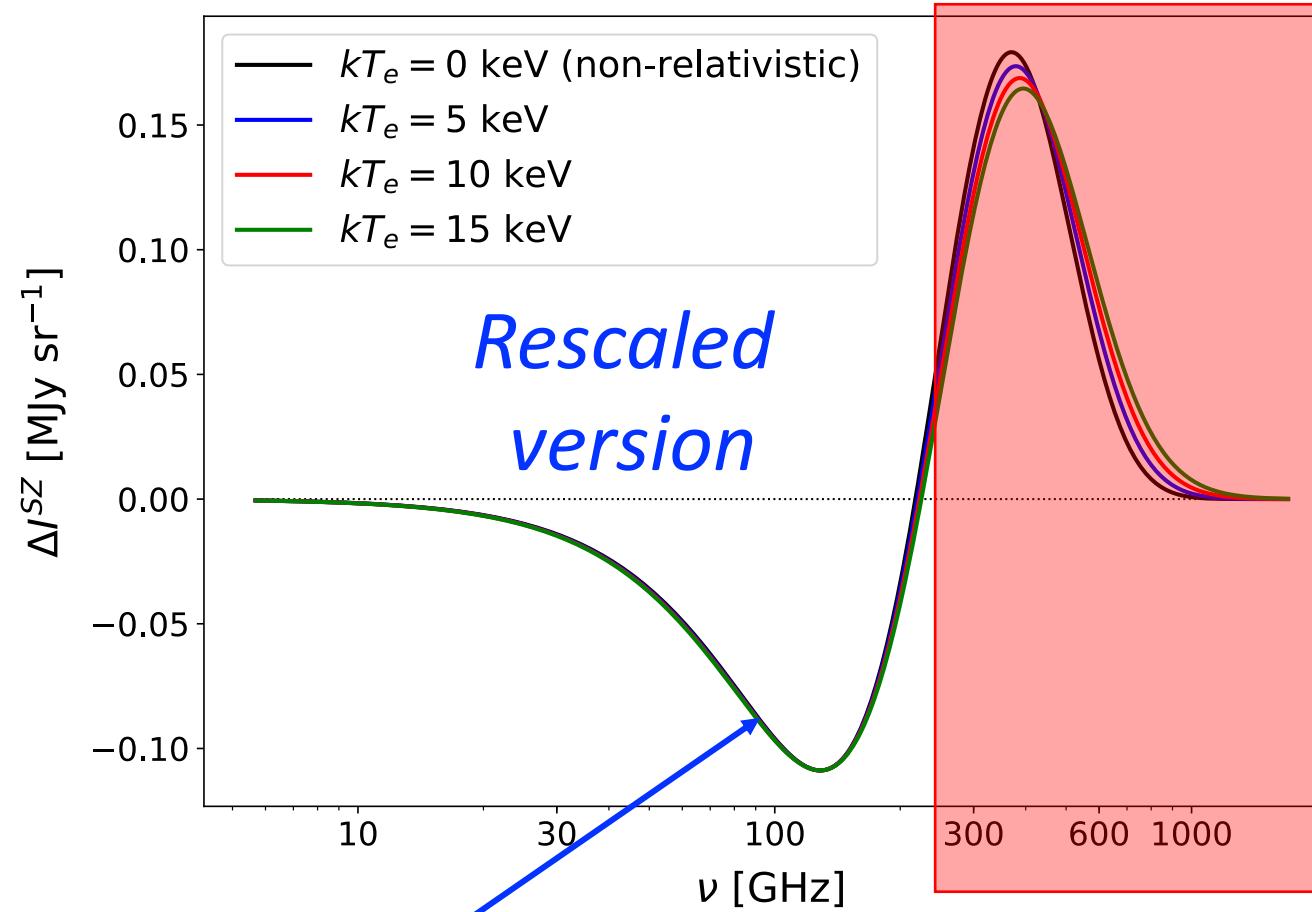


*Spectral shapes are degenerate  
at low frequency*

*(impossible to disentangle  $y$  and  $T_e$ )*

# The $y$ - $T_e$ degeneracy at low frequency

$$I_\nu^{\text{SZ}} = f(\nu, T_e(\vec{n})) y(\vec{n})$$



*High frequencies are essential to extract rSZ*

*Spectral shapes are degenerate  
at low frequency*

*(impossible to disentangle  $y$  and  $T_e$ )*

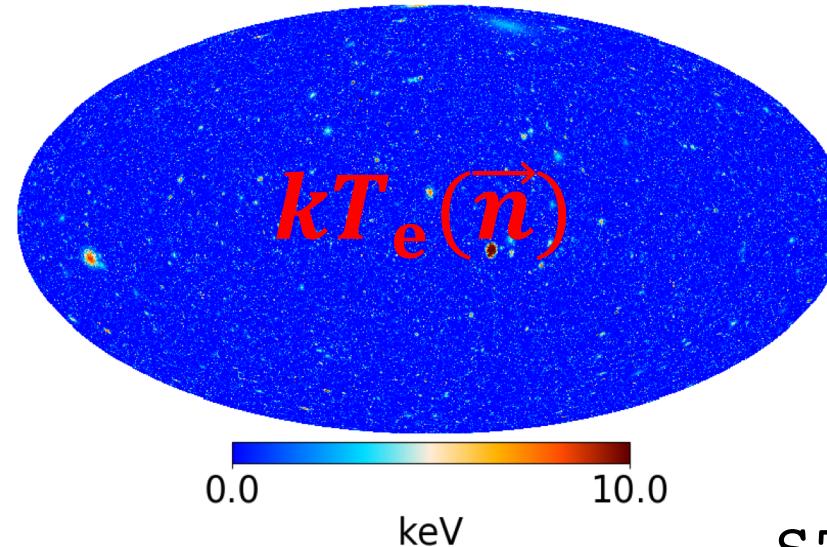
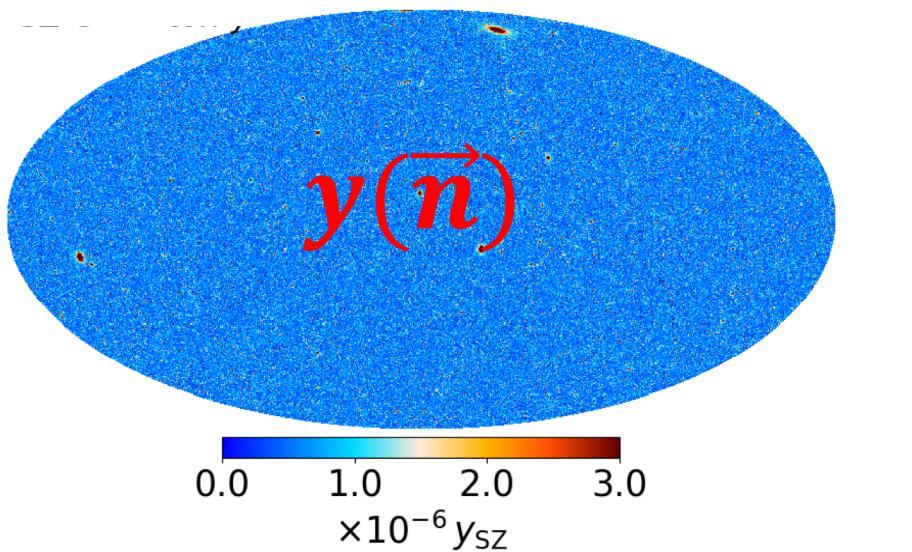
- Millimeter/submillimeter-wave, polarimetric survey of the entire sky

- 21 bands between 20 GHz and 800 GHz

- 1.4 m aperture telescope
- Diffraction limited resolution: 38' to 1'
- 13,000 transition edge sensor bolometers
- 5 year survey from L2
- 0.87  $\mu\text{K}^*\text{arcmin}$  requirement; 0.61  $\mu\text{K}^*\text{arcmin}$  goal (=CBE)



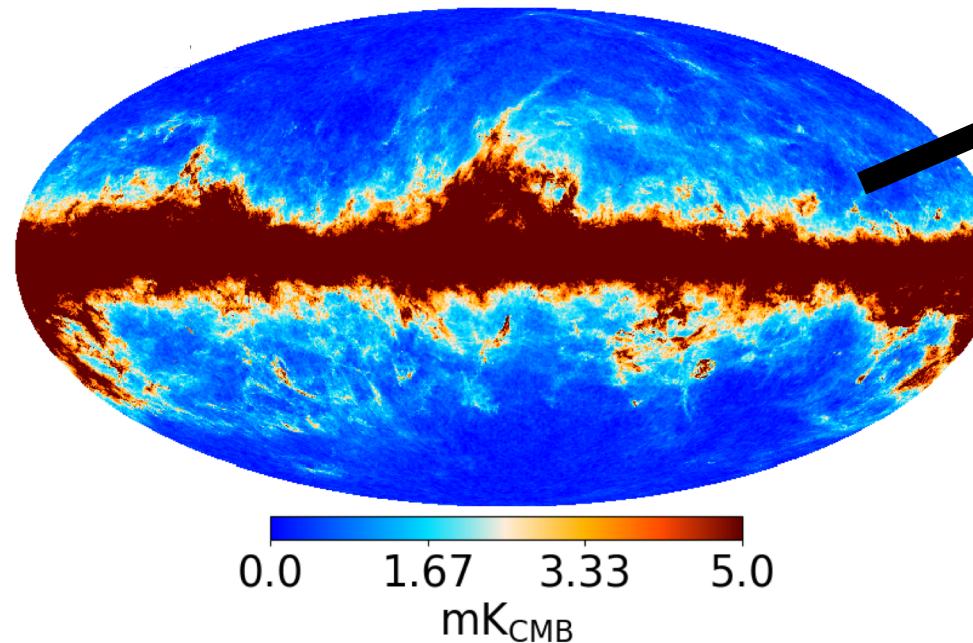
# Foreground-obscured sky simulations



rSZ maps (20 - 800 GHz):  $I_\nu^{\text{rSZ}}(\vec{n}) = f(\nu, T_e(\vec{n}))y(\vec{n})$

SZpack  
*Chluba, Nagai, Sazonov, Nelson, MNRAS 2012*

PICO sky maps  
20 - 800 GHz



$r\text{SZ}, k\text{SZ}, \text{CMB}, \text{CIB},$   
*Galactic foregrounds*  
*(dust, synchrotron,*  
*AME, free-free),*  
*noise*

# Component Separation

How to disentangle the  $y$  and  $T_e$  observables  
of the rSZ effect in sky observations?

# SZ temperature moment expansion

$$I_{\nu}^{\text{SZ}}(\vec{n}) = f(\nu, \textcolor{red}{T}_{\text{e}}(\vec{n})) y(\vec{n})$$

# SZ temperature moment expansion

$$I_{\nu}^{\text{SZ}}(\vec{n}) = f(\nu, \textcolor{red}{T}_e(\vec{n})) y(\vec{n})$$

Taylor expansion around pivot temperature  $\bar{T}_e$

$$I_{\nu}^{\text{SZ}}(\vec{n}) = \textcolor{blue}{f}(\nu, \bar{T}_e) y(\vec{n}) + \frac{\partial f(\nu, \bar{T}_e)}{\partial T_e} (T_e(\vec{n}) - \bar{T}_e) y(\vec{n}) + \mathcal{O}(T_e^2)$$

# SZ temperature moment expansion

$$I_{\nu}^{\text{SZ}}(\vec{n}) = f(\nu, \mathbf{T}_e(\vec{n})) y(\vec{n})$$

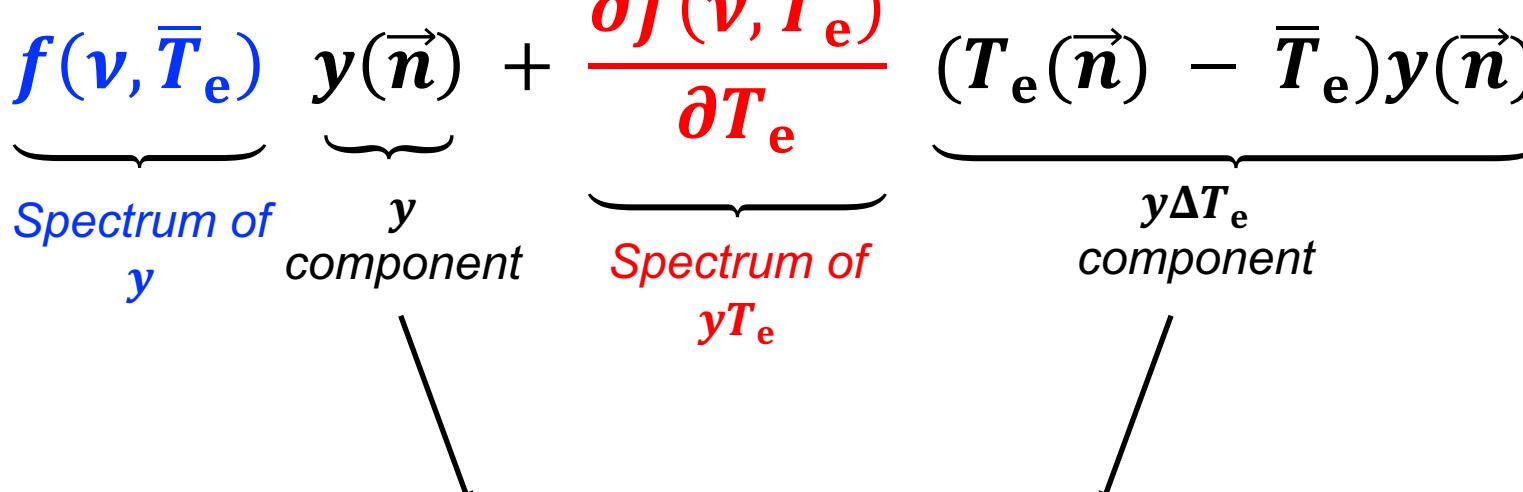
Taylor expansion around pivot temperature  $\bar{T}_e$

$$I_{\nu}^{\text{SZ}}(\vec{n}) = \underbrace{f(\nu, \bar{T}_e)}_{\substack{\text{Spectrum of} \\ y}} \underbrace{y(\vec{n})}_{\substack{y \\ \text{component}}} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\substack{\text{Spectrum of} \\ yT_e}} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{\substack{y\Delta T_e \\ \text{component}}} + \mathcal{O}(T_e^2)$$

# SZ temperature moment expansion

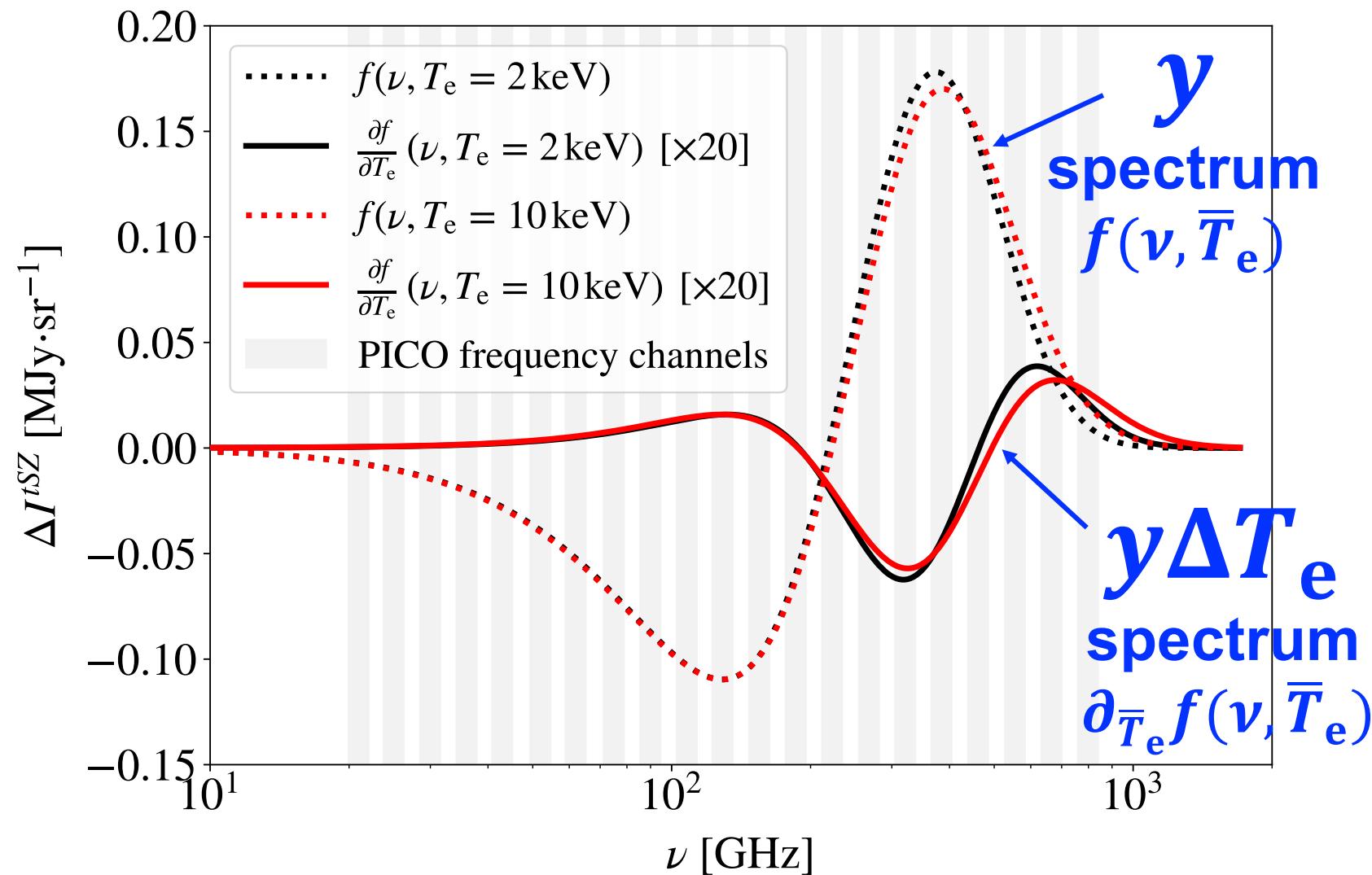
$$I_{\nu}^{\text{SZ}}(\vec{n}) = f(\nu, \mathbf{T}_e(\vec{n})) y(\vec{n})$$

Taylor expansion around pivot temperature  $\bar{T}_e$

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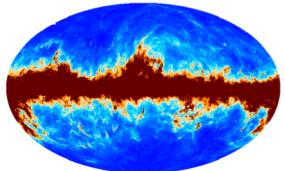
Two distinct components of emission,  $y$  and  $y\Delta T_e$ ,  
with different spectral signatures!

# Two spectral components of the rSZ effect



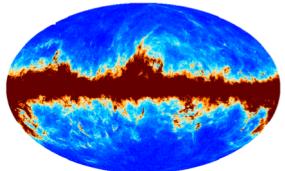
*It is possible in principle to disentangle  $y$  and  $y\Delta T_e$  through multi-frequency observations and component separation methods*

# rSZ component separation



$$d(\nu, \vec{n}) = \underbrace{\mathbf{f}(\nu, \bar{T}_e)}_{\text{spectrum of } y(\vec{n})} \underbrace{y(\vec{n})}_{y(\vec{n})} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e(\vec{n})} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{y\Delta T_e(\vec{n})} + \underbrace{N(\nu, \vec{n})}_{\text{foregrounds} + \text{noise}}$$

# rSZ component separation

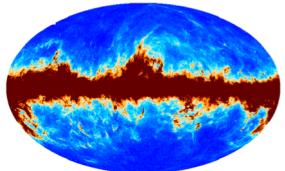


$$d(\nu, \vec{n}) = \underbrace{f(\nu, \bar{T}_e)}_{\text{spectrum of } y(\vec{n})} \underbrace{y(\vec{n})}_{y(\vec{n})} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e(\vec{n})} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{y\Delta T_e(\vec{n})} + \underbrace{N(\nu, \vec{n})}_{\text{foregrounds} + \text{noise}}$$

Component separation with the Constrained ILC method ([Remazeilles et al MNRAS 2011](#))

$$\widehat{y\Delta T_e}(\vec{n}) = \sum_{\nu} w(\nu) d(\nu, \vec{n}) \quad \text{such that} \quad \left\{ \begin{array}{l} \left\langle (\widehat{y\Delta T_e})^2 \right\rangle \text{ of minimum variance} \\ \sum_{\nu} w(\nu) \frac{\partial f(\nu, \bar{T}_e)}{\partial T_e} = 1 \\ \sum_{\nu} w(\nu) f(\nu, \bar{T}_e) = 0 \end{array} \right.$$

# rSZ component separation



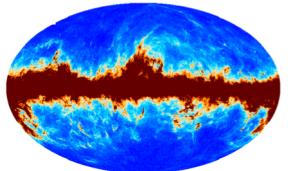
$$d(\nu, \vec{n}) = \underbrace{f(\nu, \bar{T}_e)}_{\text{spectrum of } y(\vec{n})} \underbrace{y(\vec{n})}_{y(\vec{n})} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e(\vec{n})} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{y\Delta T_e(\vec{n})} + \underbrace{N(\nu, \vec{n})}_{\text{foregrounds} + \text{noise}}$$

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Guarantees the conservation of the signal of interest  $y\Delta T_e$

# rSZ component separation



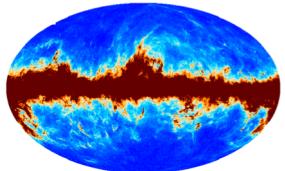
$$d(\nu, \vec{n}) = \underbrace{f(\nu, \bar{T}_e)}_{\text{spectrum of } y(\vec{n})} \underbrace{y(\vec{n})}_{y(\vec{n})} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e(\vec{n})} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{y\Delta T_e(\vec{n})} + \underbrace{N(\nu, \vec{n})}_{\text{foregrounds} + \text{noise}}$$

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Guarantees the cancellation of  $y$  residuals in the  $y\Delta T_e$  map

# rSZ component separation



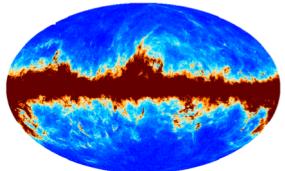
$$d(\nu, \vec{n}) = \underbrace{f(\nu, \bar{T}_e)}_{\text{spectrum of } y(\vec{n})} \underbrace{y(\vec{n})}_{y(\vec{n})} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e(\vec{n})} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{y\Delta T_e(\vec{n})} + \underbrace{N(\nu, \vec{n})}_{\text{foregrounds} + \text{noise}}$$

Component separation with the Constrained ILC method ([Remazeilles et al MNRAS 2011](#))

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Guarantees the mitigation of foregrounds and noise

# rSZ component separation



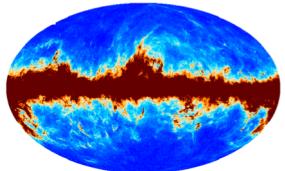
$$d(\nu, \vec{n}) = \underbrace{\mathbf{f}(\nu, \bar{T}_e)}_{\text{spectrum of } y(\vec{n})} \underbrace{y(\vec{n})}_{y(\vec{n})} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e(\vec{n})} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{y\Delta T_e(\vec{n})} + \underbrace{N(\nu, \vec{n})}_{\text{foregrounds} + \text{noise}}$$

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$$\Rightarrow w = \frac{(\mathbf{f}^T \mathbf{C}^{-1} \mathbf{f}) \partial \mathbf{f}^T \mathbf{C}^{-1} - (\partial \mathbf{f}^T \mathbf{C}^{-1} \mathbf{f}) \mathbf{f}^T \mathbf{C}^{-1}}{(\partial \mathbf{f}^T \mathbf{C}^{-1} \partial \mathbf{f})(\mathbf{f}^T \mathbf{C}^{-1} \mathbf{f}) - (\partial \mathbf{f}^T \mathbf{C}^{-1} \mathbf{f})^2} \quad \text{where} \quad \mathbf{C}_{\nu\nu'} \equiv \langle d(\nu, \vec{n}) d(\nu', \vec{n}) \rangle$$

# rSZ component separation



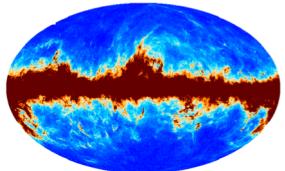
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Component separation with the Constrained ILC method ([Remazeilles et al MNRAS 2011](#))

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$$\Rightarrow \widehat{y\Delta T_e}(\vec{n}) = (w \cdot \mathbf{f}) y(\vec{n}) + (w \cdot \partial_{T_e} \mathbf{f})(T_e(\vec{n}) - \bar{T}_e) y(\vec{n}) + w \cdot N$$

# rSZ component separation



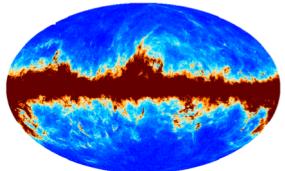
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Component separation with the Constrained ILC method ([Remazeilles et al MNRAS 2011](#))

$$\widehat{y\Delta T_e}(\vec{n}) = \sum_{\nu} w(\nu) d(\nu, \vec{n}) \quad \text{such that} \quad \begin{cases} \langle (\widehat{y\Delta T_e})^2 \rangle \text{ of minimum variance} \\ \sum_{\nu} w(\nu) \frac{\partial f(\nu, \bar{T}_e)}{\partial T_e} = 1 \\ \sum_{\nu} w(\nu) f(\nu, \bar{T}_e) = 0 \end{cases}$$

$$\Rightarrow \widehat{y\Delta T_e}(\vec{n}) = \underbrace{(w \cdot \mathbf{f}) y(\vec{n})}_{=0} + \underbrace{(w \cdot \partial_{T_e} \mathbf{f})(T_e(\vec{n}) - \bar{T}_e)}_{=1} y(\vec{n}) + \underbrace{w \cdot N}_{\text{minimised}}$$

# rSZ component separation



$$d(\nu, \vec{n}) = \underbrace{f(\nu, \bar{T}_e)}_{\text{spectrum of } y(\vec{n})} \underbrace{y(\vec{n})}_{y(\vec{n})} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e(\vec{n})} \underbrace{(T_e(\vec{n}) - \bar{T}_e)y(\vec{n})}_{y\Delta T_e(\vec{n})} + \underbrace{N(\nu, \vec{n})}_{\text{foregrounds} + \text{noise}}$$

Component separation with the Constrained ILC method ([Remazeilles et al MNRAS 2011](#))

$$\widehat{y\Delta T_e}(\vec{n}) = \sum_{\nu} w(\nu) d(\nu, \vec{n}) \quad \text{such that} \quad \left\{ \begin{array}{l} \left\langle (\widehat{y\Delta T_e})^2 \right\rangle \text{ of minimum variance} \\ \sum_{\nu} w(\nu) \frac{\partial f(\nu, \bar{T}_e)}{\partial T_e} = 1 \\ \sum_{\nu} w(\nu) f(\nu, \bar{T}_e) = 0 \end{array} \right.$$

$$\Rightarrow \widehat{y\Delta T_e}(\vec{n}) = \underbrace{(T_e(\vec{n}) - \bar{T}_e) y(\vec{n})}_{\text{red underline}} + w \cdot N \quad \textcolor{red}{T_e\text{-modulated } y\text{-map!}}$$

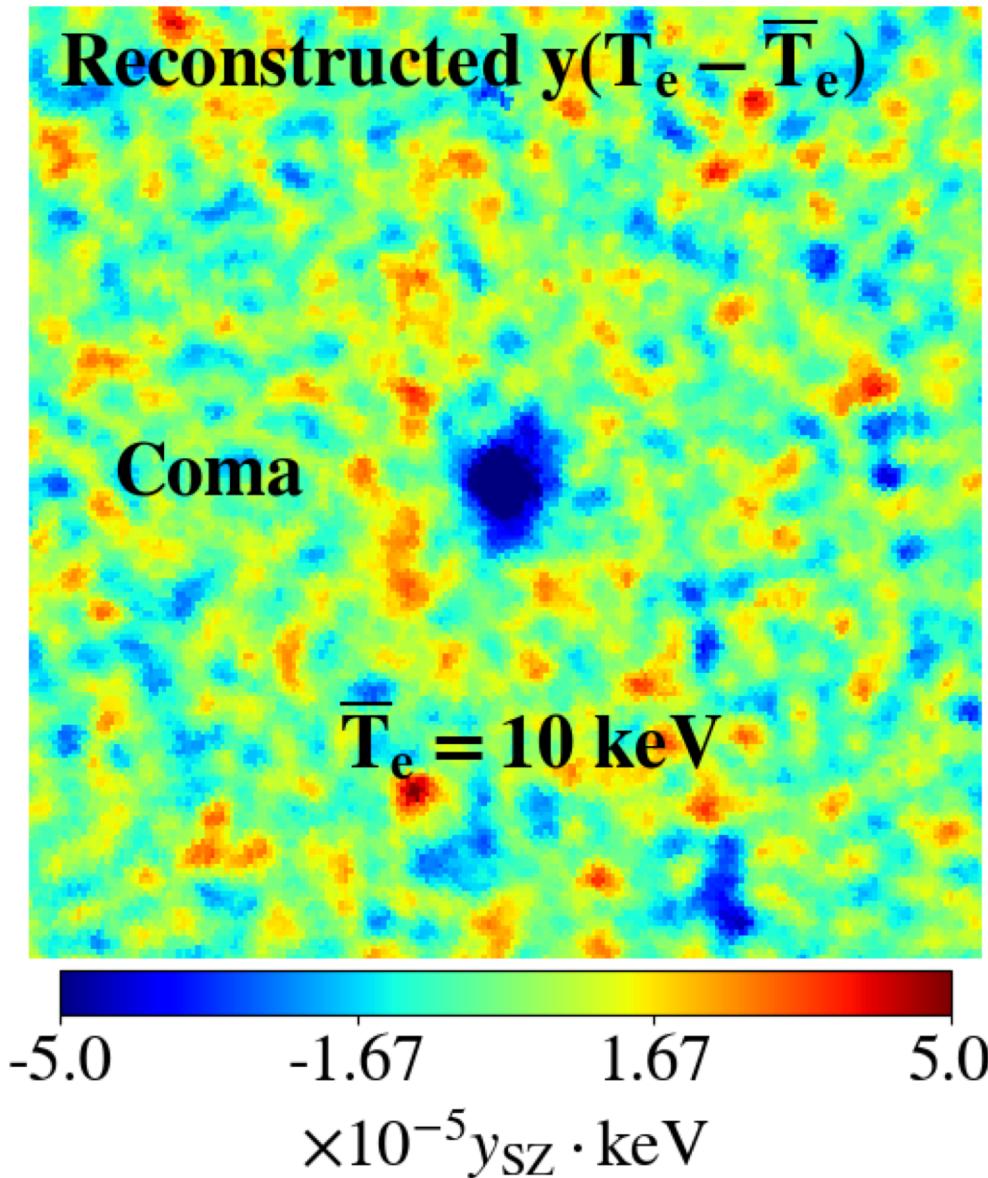
# Why is this new SZ observable so interesting?

$$y\Delta T_e(\vec{n}) \equiv y(\vec{n})(T_e(\vec{n}) - \bar{T}_e)$$

Changing the pivot temperature  $\bar{T}_e$  in the analysis allows us to conduct a real **temperature spectroscopy** of the cluster:

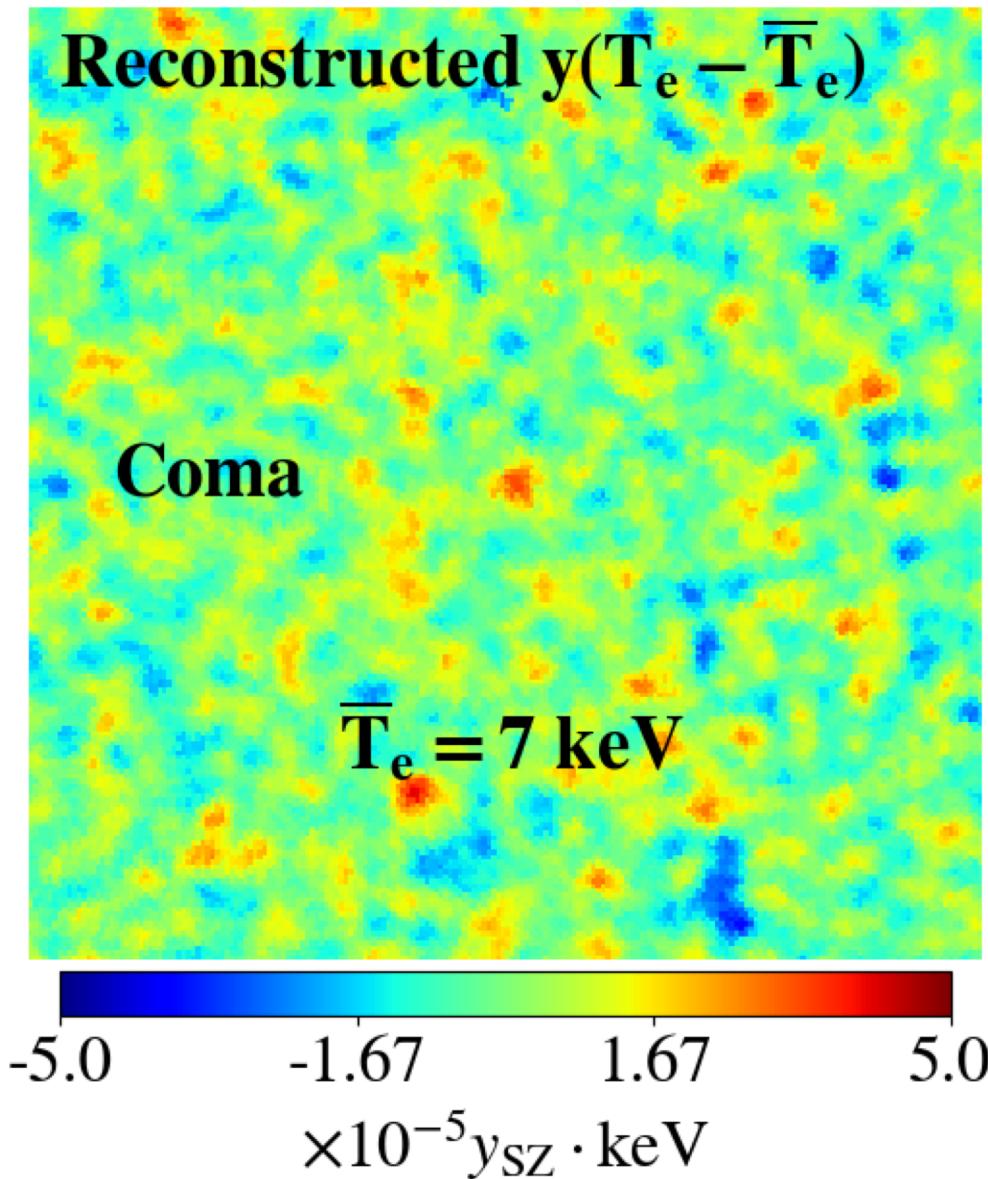
- Decrement if actual temperature  $T_e(\vec{n}) < \bar{T}_e$
- Increment if actual temperature  $T_e(\vec{n}) > \bar{T}_e$
- Null if actual temperature  $T_e(\vec{n}) \simeq \bar{T}_e$

# Cluster spectroscopy across temperature



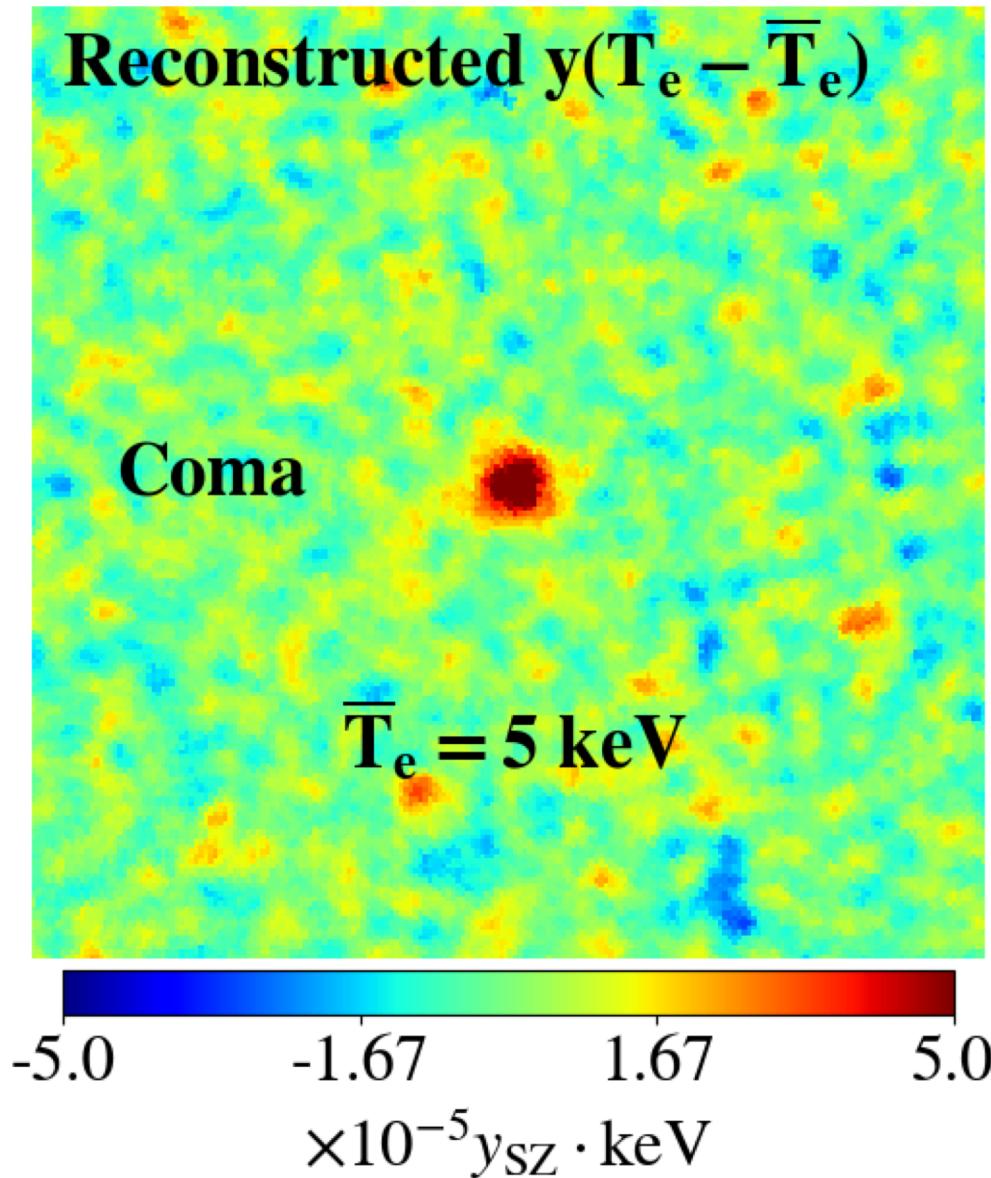
$$\bar{T}_e = 10 \text{ keV}$$

# Cluster spectroscopy across temperature



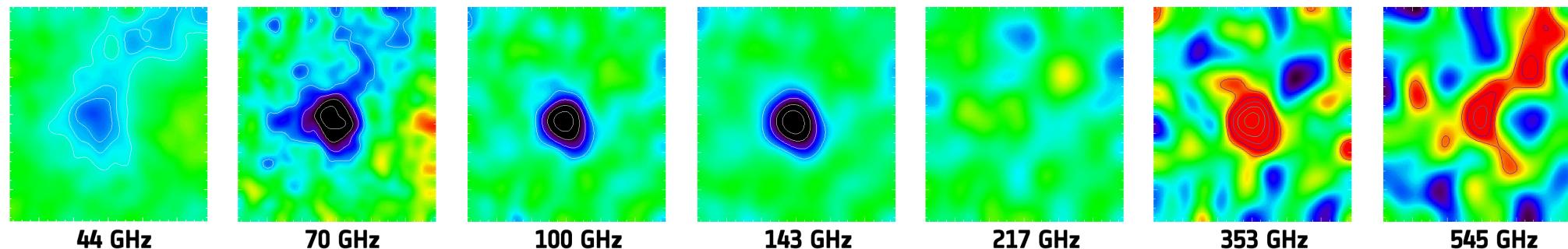
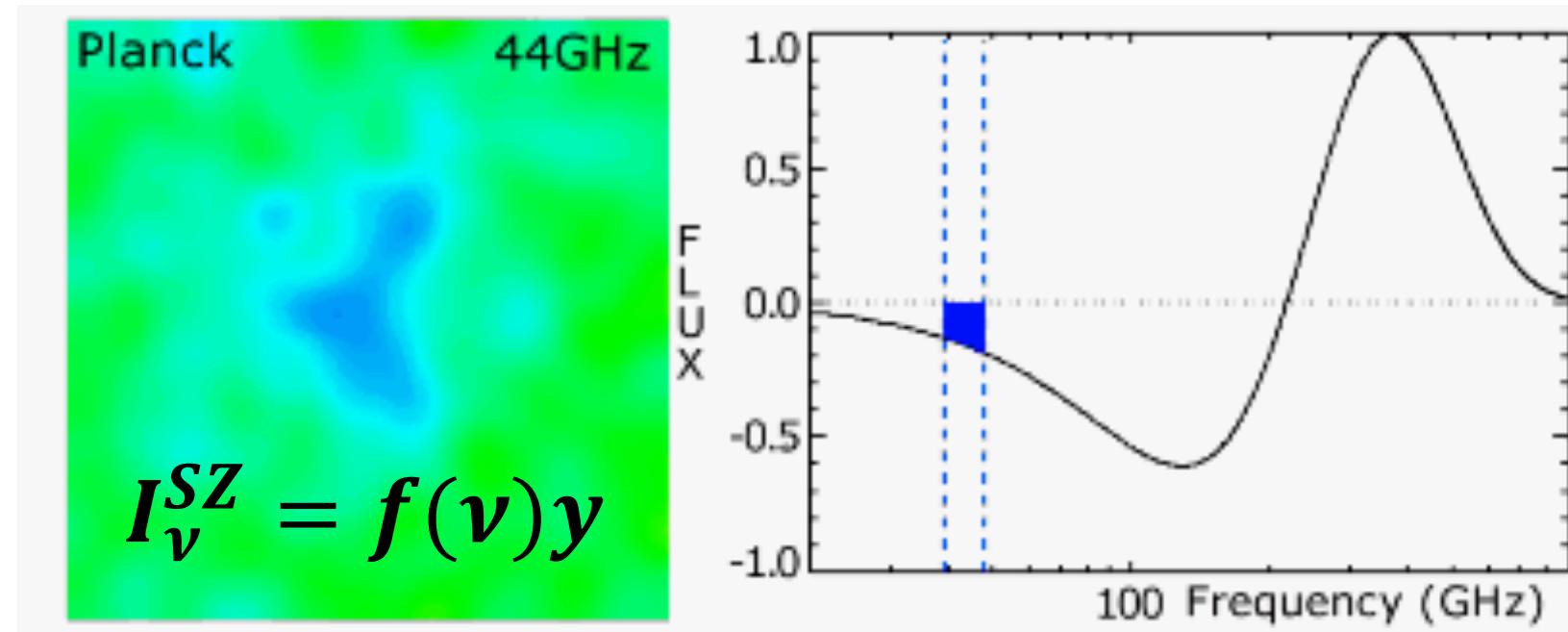
$$\bar{T}_e = 7 \text{ keV}$$

# Cluster spectroscopy across temperature



$$\bar{T}_e = 5 \text{ keV}$$

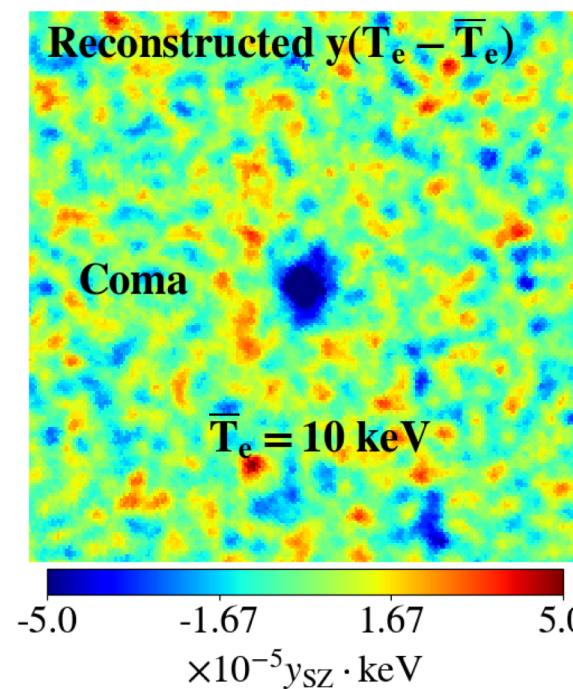
# “First SZ revolution”: cluster spectroscopy across frequency



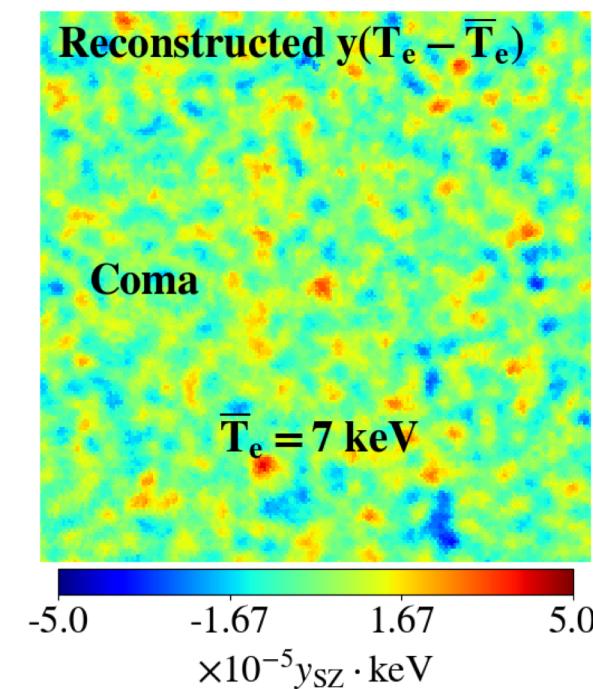
Credit: ESA/Planck Collaboration

# “Second SZ revolution”: cluster spectroscopy across temperature

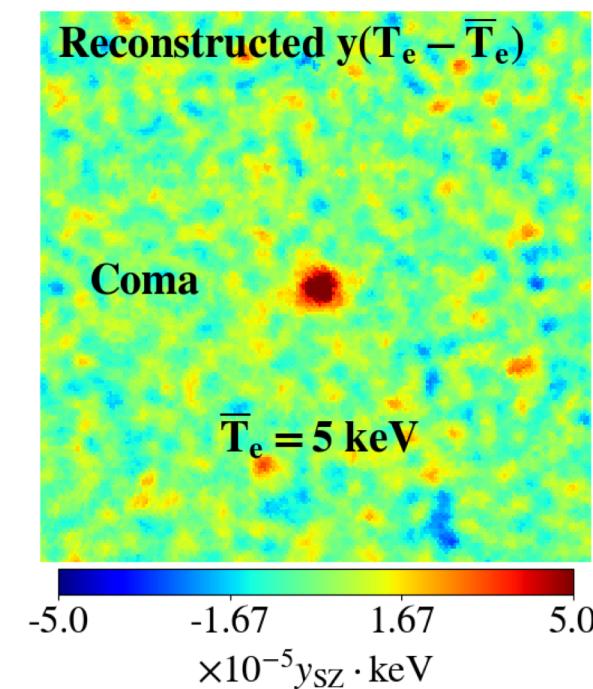
Recovered  $y(T_e - \bar{T}_e)$ -map for different pivots



colder than 10 keV  
(decrement)

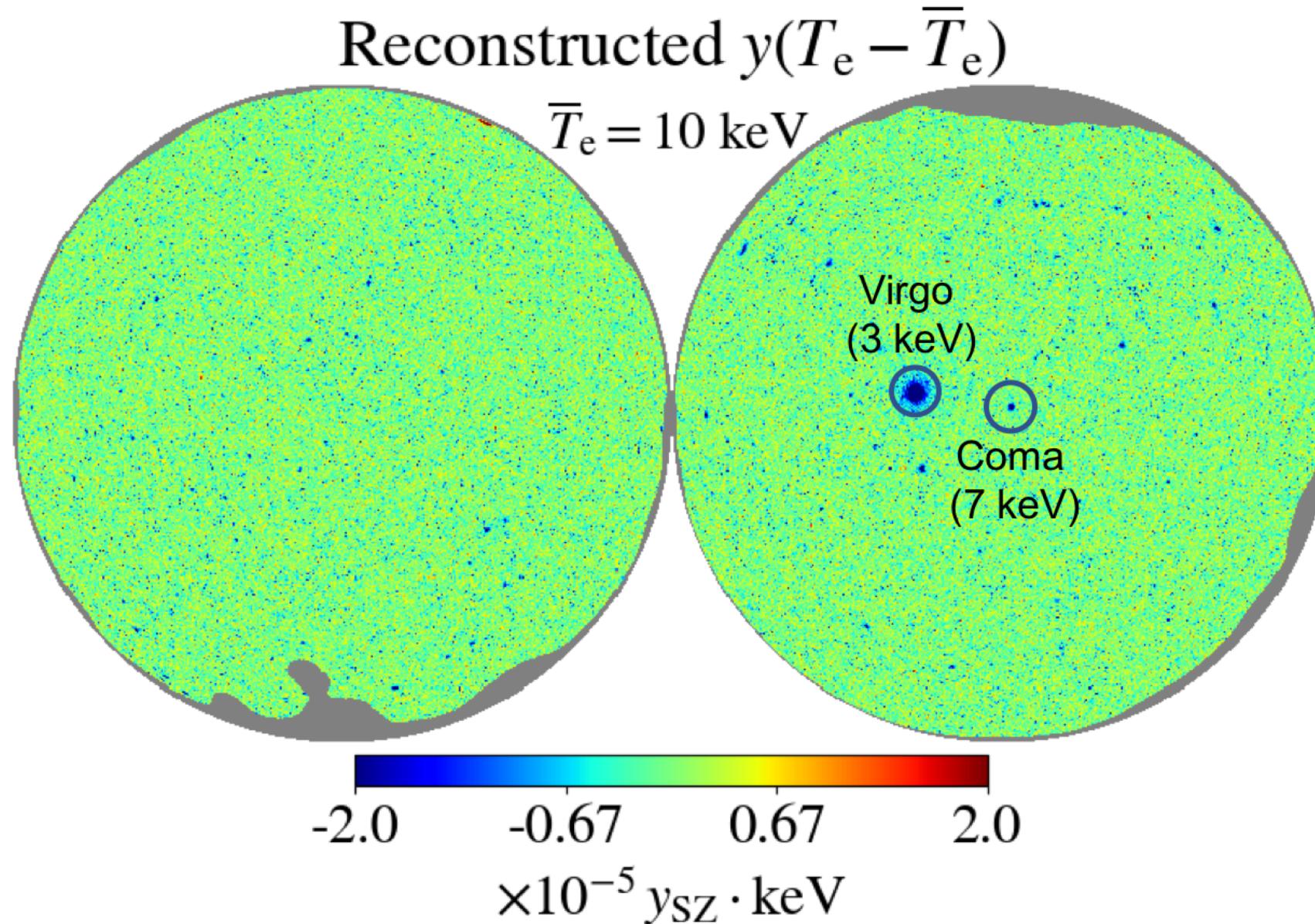


closer to 7 keV  
(null)

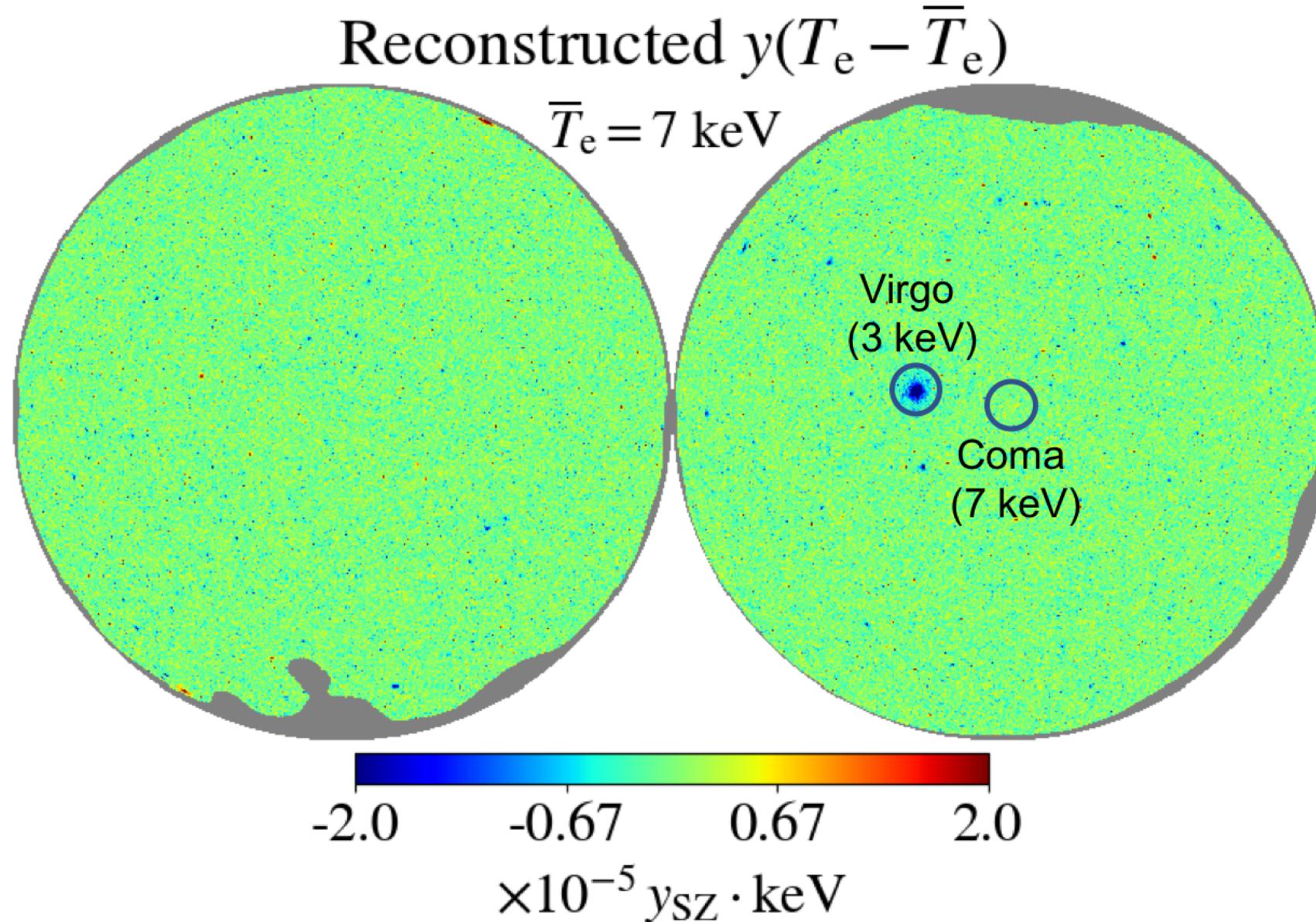


hotter than 5 keV  
(increment)

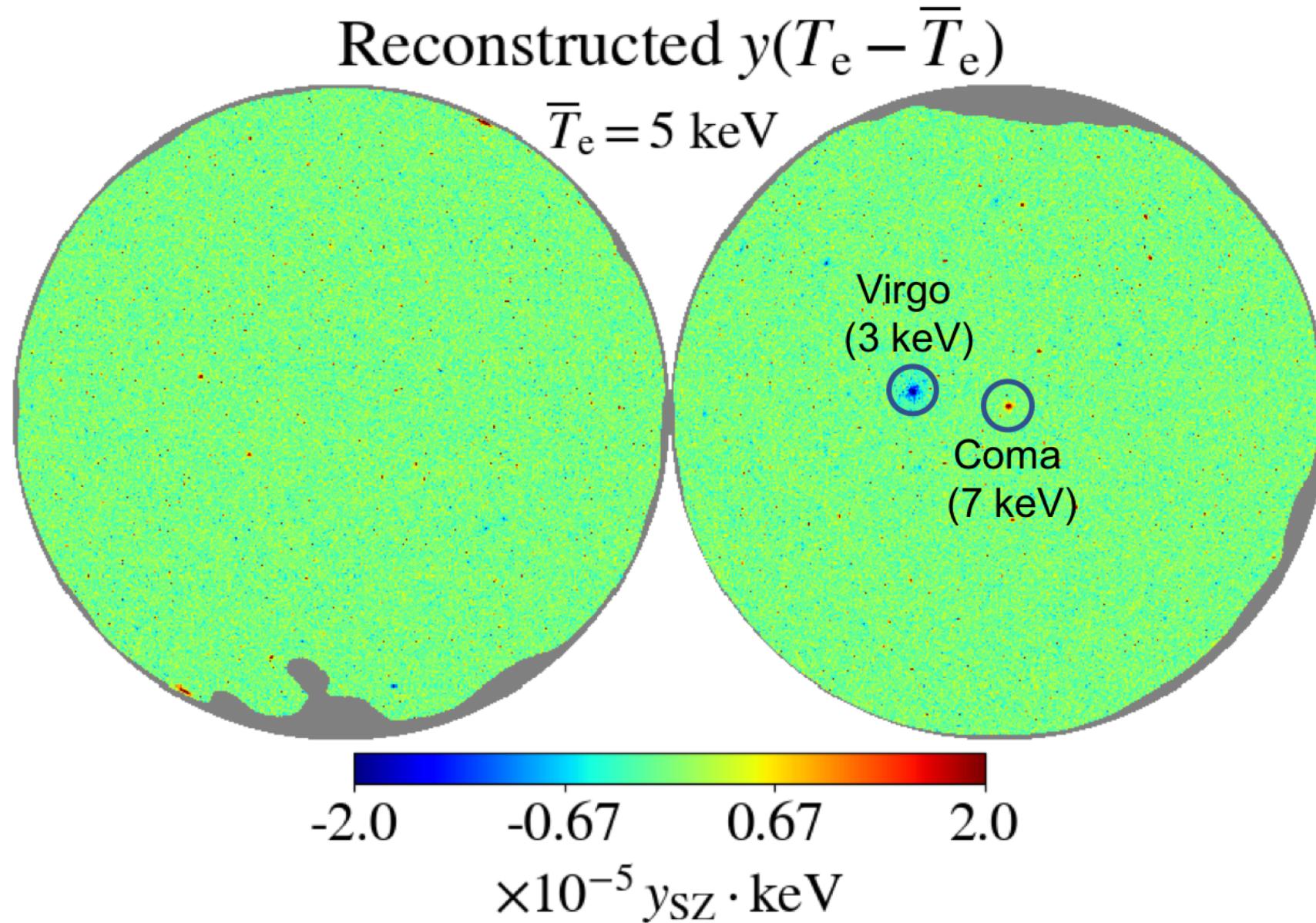
# Full-sky temperature spectroscopy



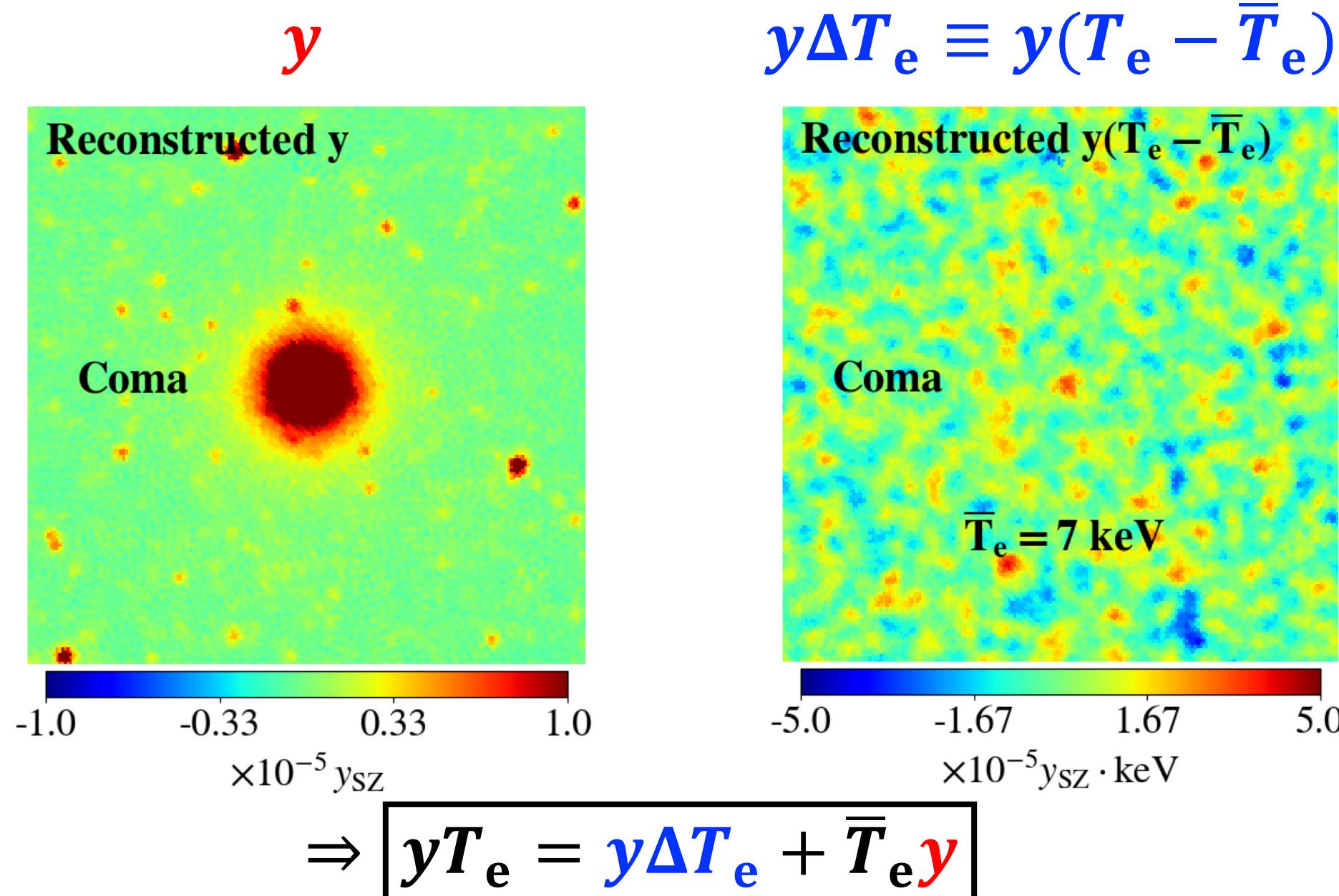
# Full-sky temperature spectroscopy



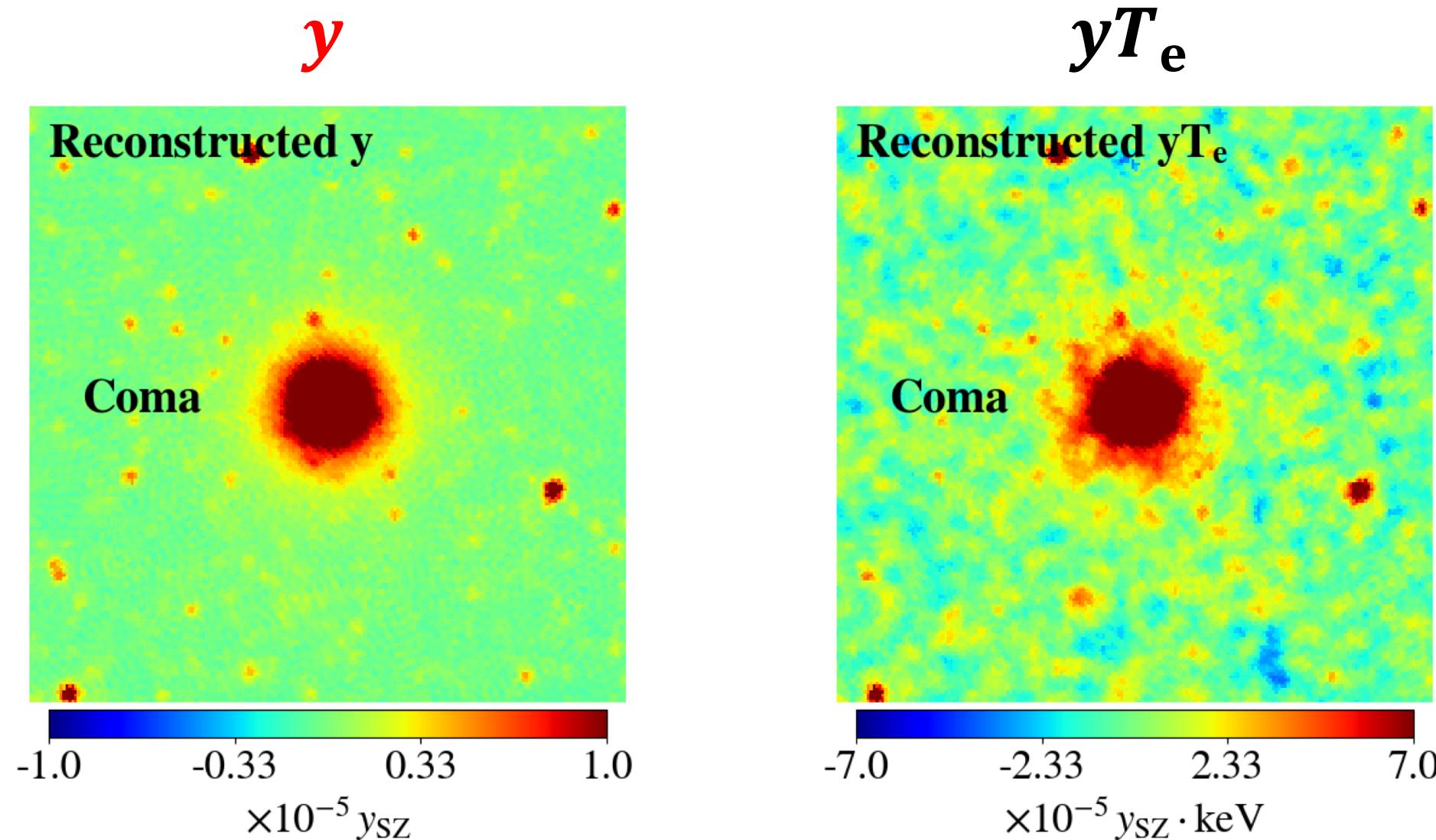
# Full-sky temperature spectroscopy



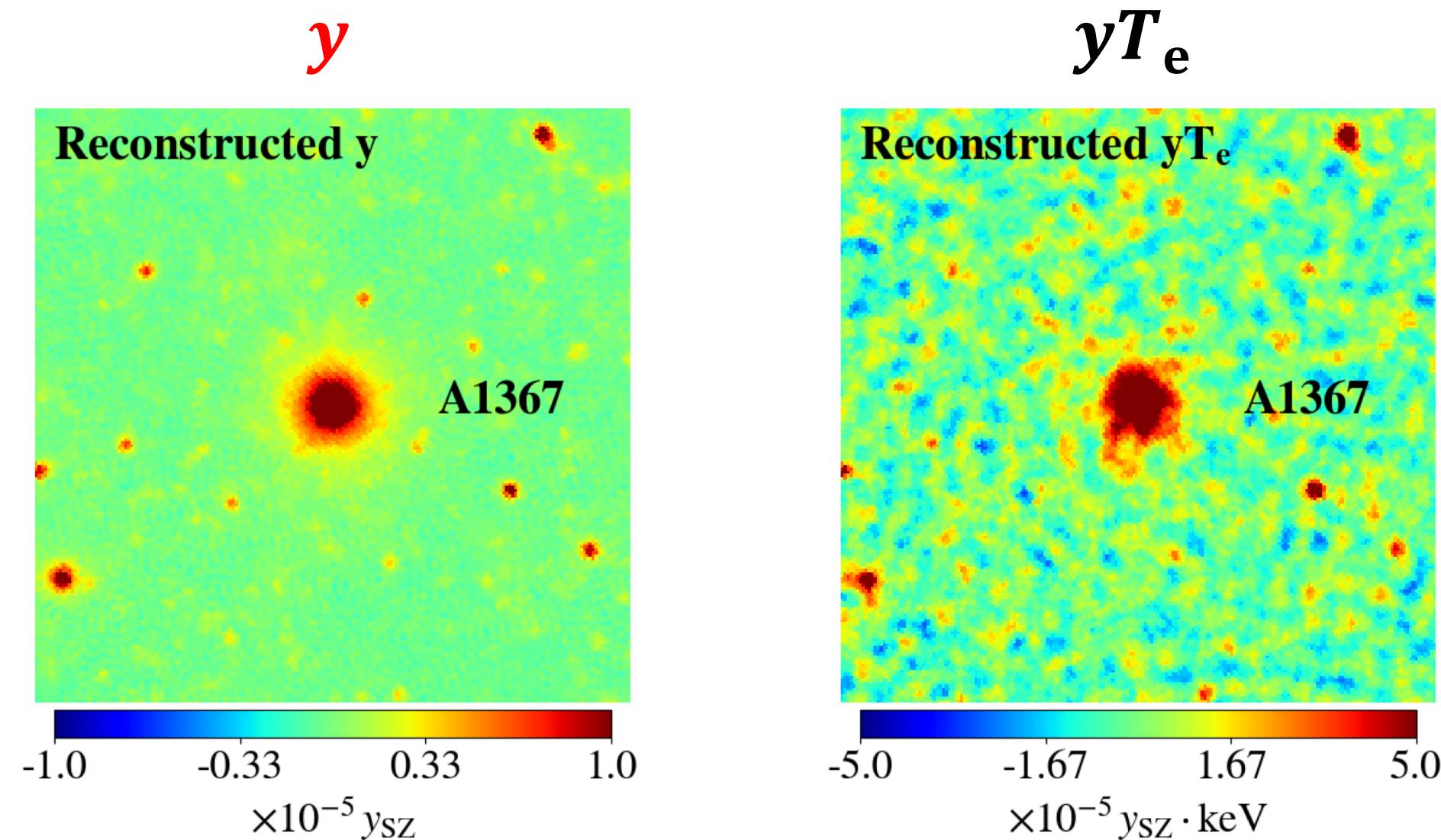
# Reconstructed rSZ components



# Mapping the $y$ and $yT_e$ components



# Mapping the $y$ and $yT_e$ components

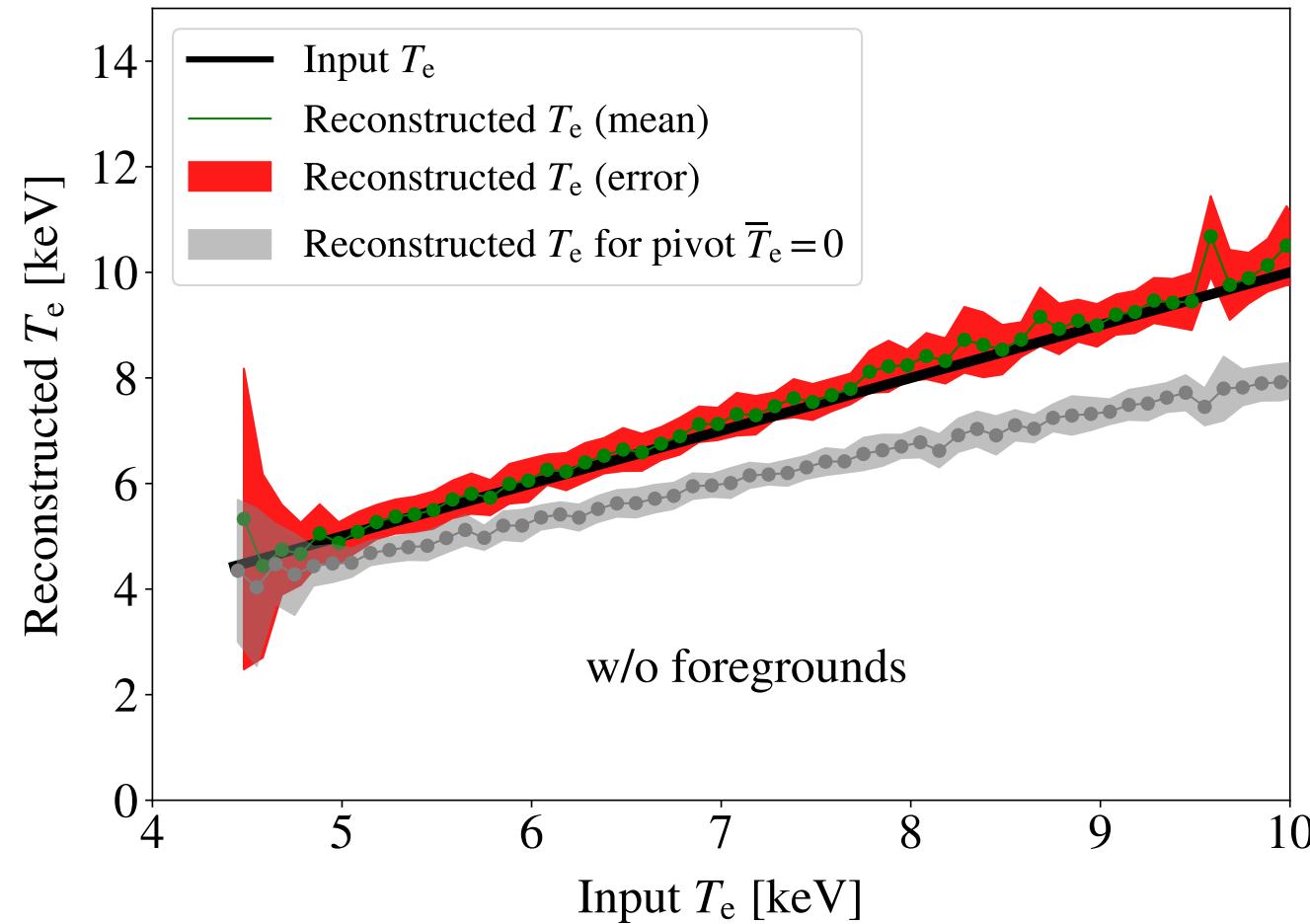


# Mapping cluster temperatures $T_e$ across the entire sky

$$T_e^y[R_{500}] = \frac{\langle (yT_e)(\vec{n}) \rangle_{|\vec{n}-\vec{n}_c| \leq R_{500}}}{\langle y(\vec{n}) \rangle_{|\vec{n}-\vec{n}_c| \leq R_{500}}}$$

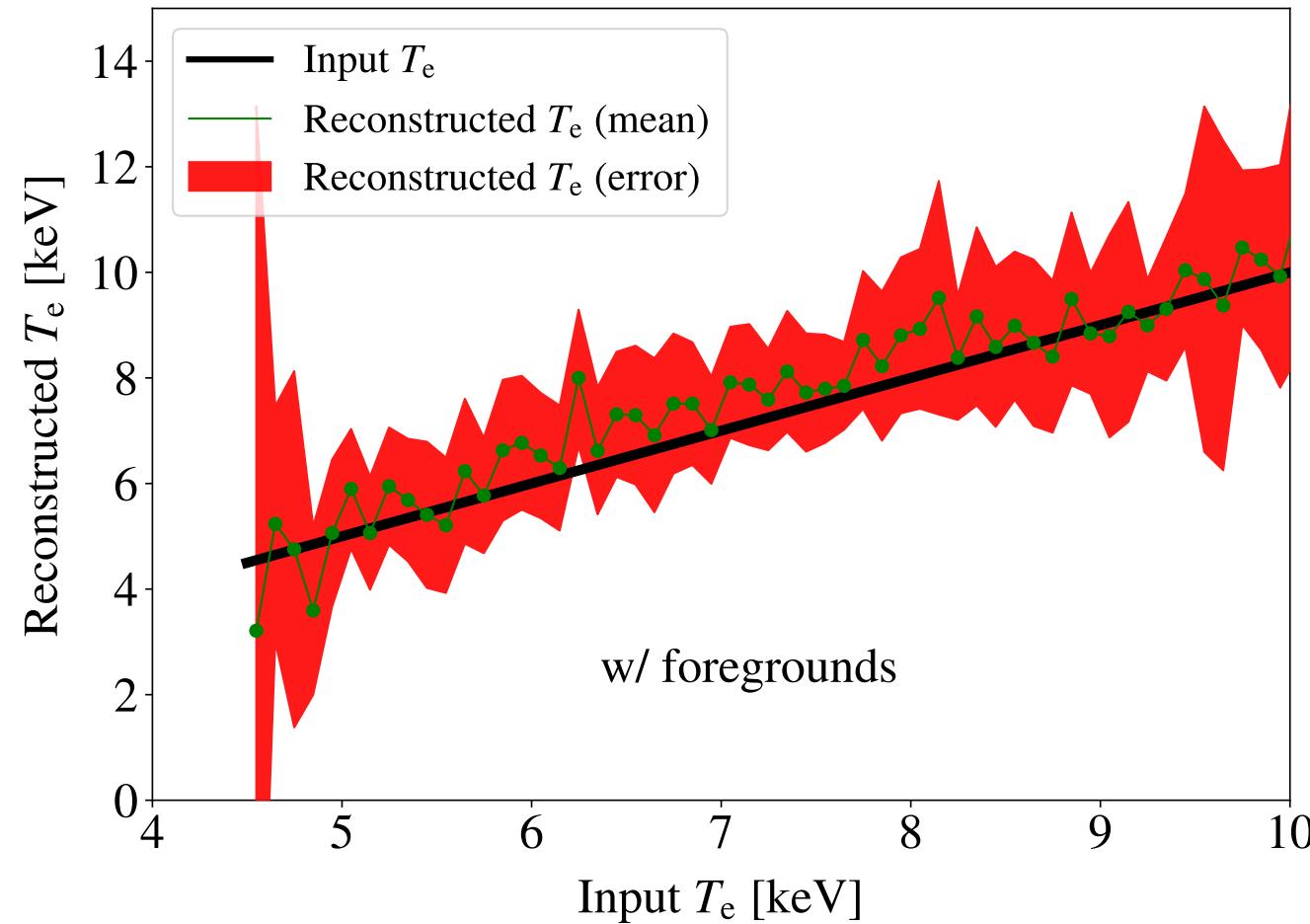
$y$ -weighted average cluster temperature over  $R_{500}$

# Recovered electron temperatures $T_e$ of clusters across the entire sky



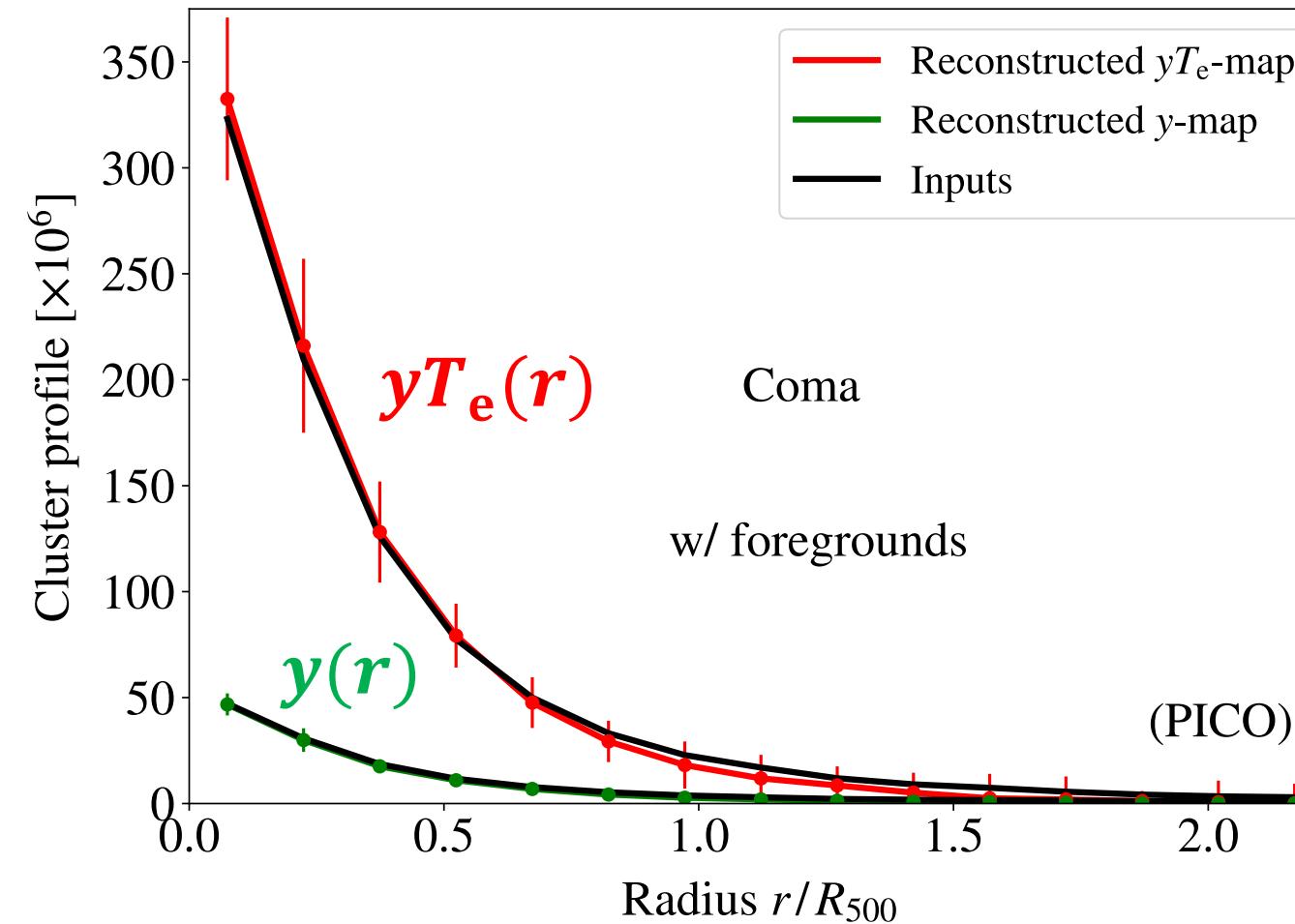
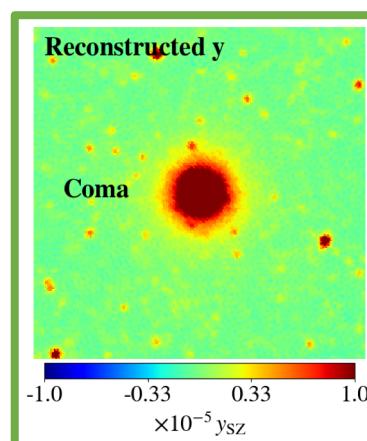
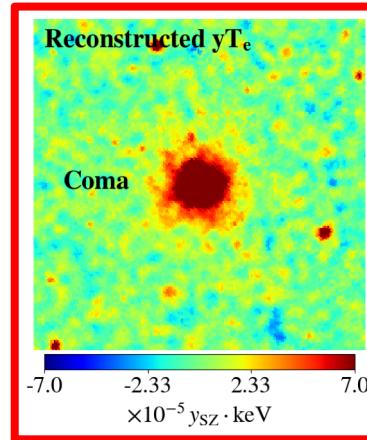
The recovered rSZ temperatures offer a new proxy for determining cluster masses without relying on X-rays

# Recovered electron temperatures $T_e$ of clusters across the entire sky



The recovered rSZ temperatures offer a new proxy for determining cluster masses without relying on X-rays

# Reconstructed cluster profiles

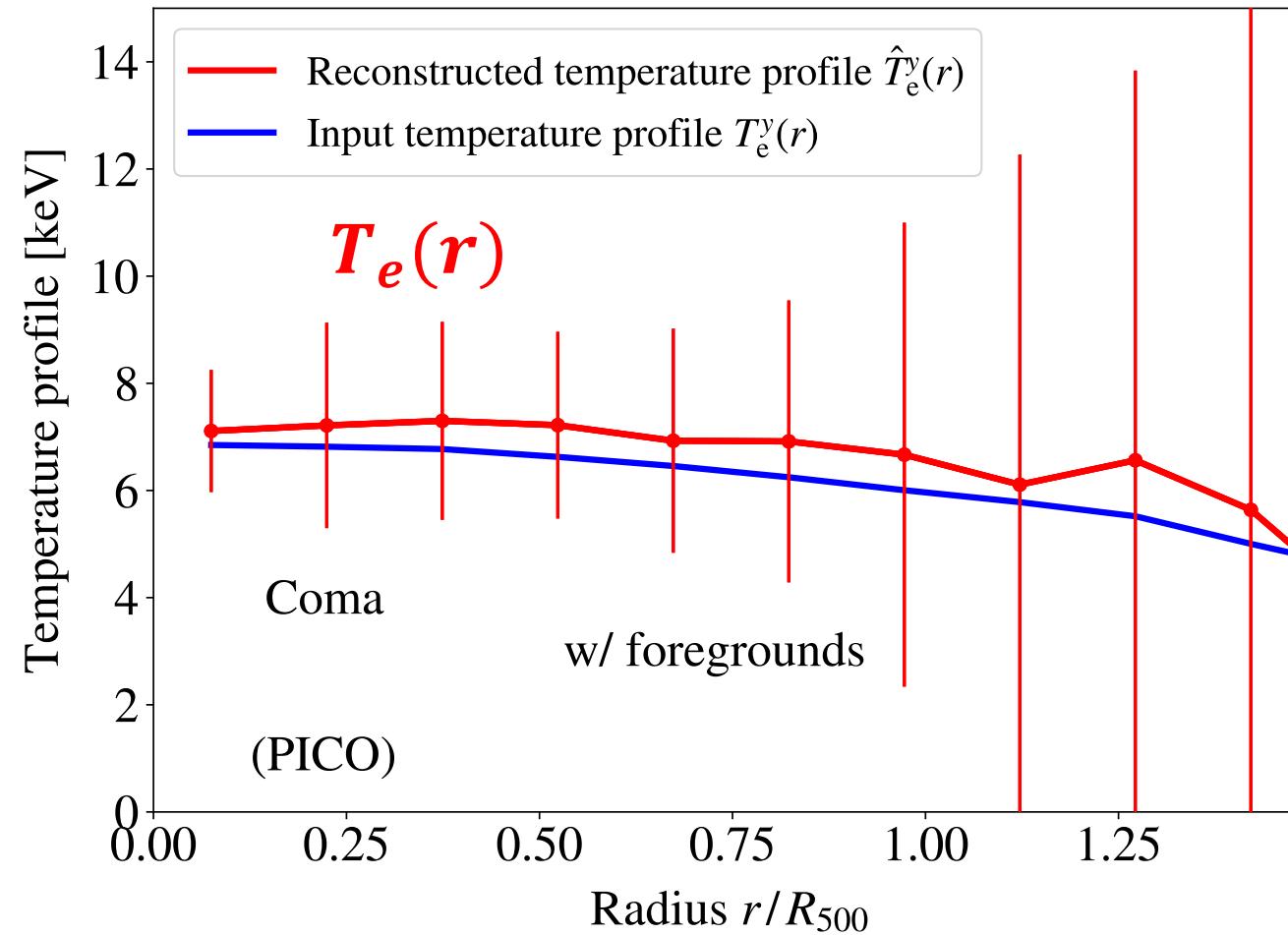


# Temperature profiles of clusters

$$T_e^y(r) = \frac{(yT_e)(r)}{y(r)}$$

$y$ -weighted temperature profile

# Reconstructed cluster temperature profile



$$\langle T_e \rangle_{R_{500}} = (7.1 \pm 0.7) \text{ keV}$$

10 $\sigma$  measurement with PICO

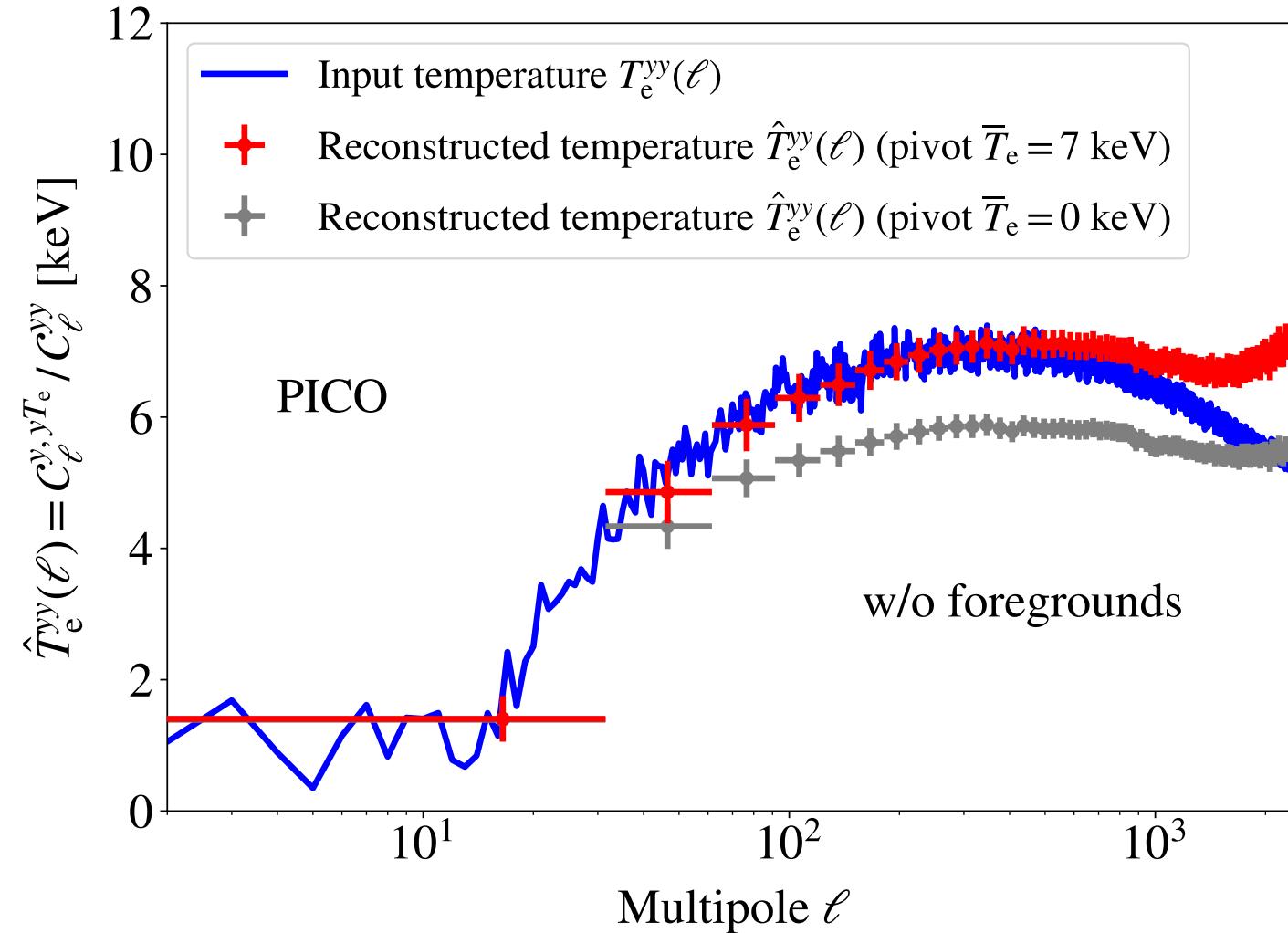
# Electron temperature power spectrum

$$T_e^{yy}(\ell) = \frac{\langle (yT_e)_{\ell m} y_{\ell m}^* \rangle}{\langle y_{\ell m} y_{\ell m}^* \rangle} = \frac{C_\ell^{y,yT_e}}{C_\ell^{yy}}$$

$y^2$ -weighted average temperature over the full sky across multipoles

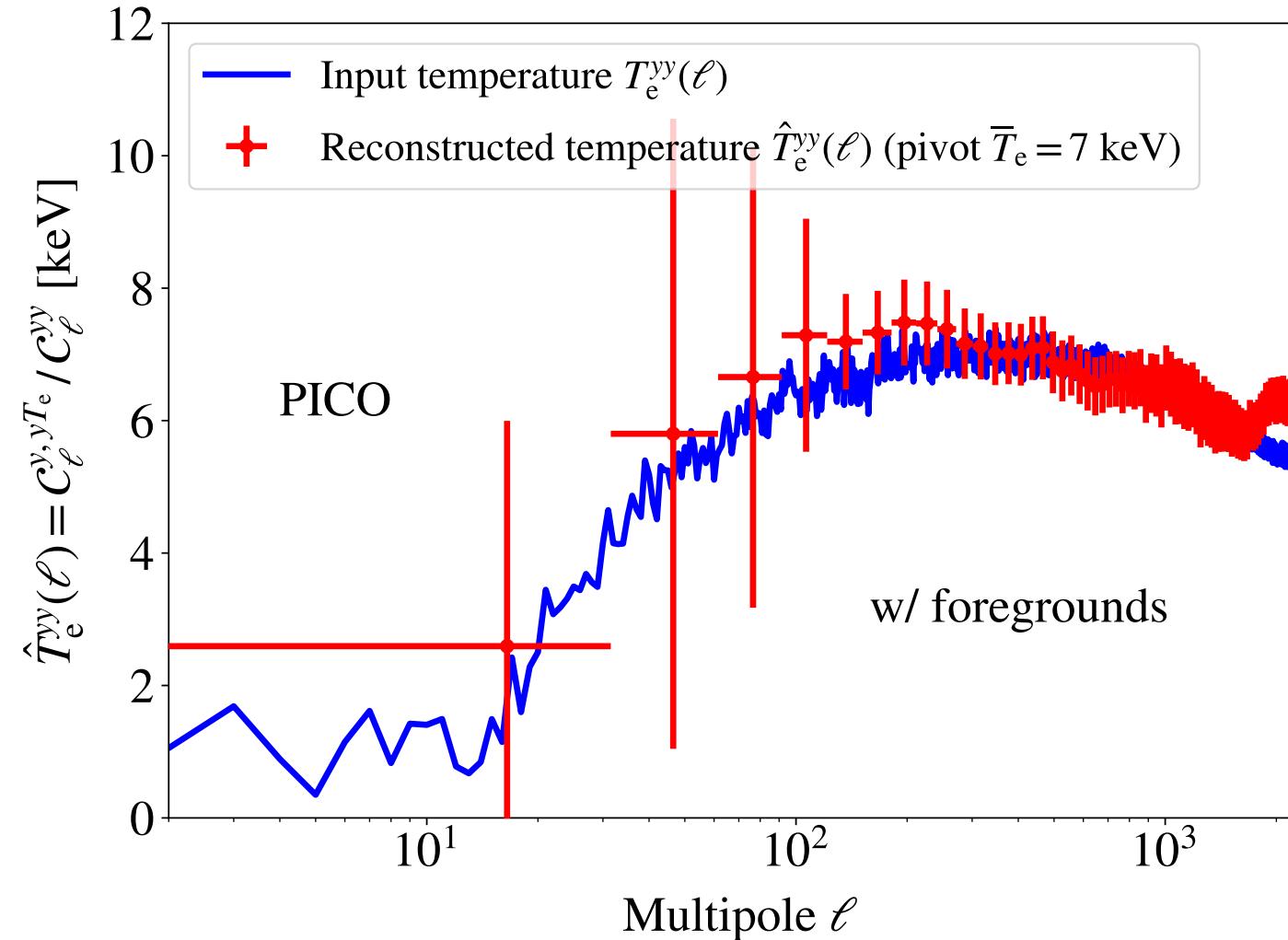
*Ratio of cross-power spectrum between the  $y$ - and  $yT_e$ -maps  
and auto-power spectrum of the  $y$ -map*

# Electron temperature power spectrum $T_e^{yy}(\ell)$



$T_e^{yy}(\ell)$  provides a new map-based observable, complementing the  $y$ -map power spectrum  $C_\ell^{yy}$ , to constrain cosmological parameters

# Electron temperature power spectrum $T_e^{yy}(\ell)$

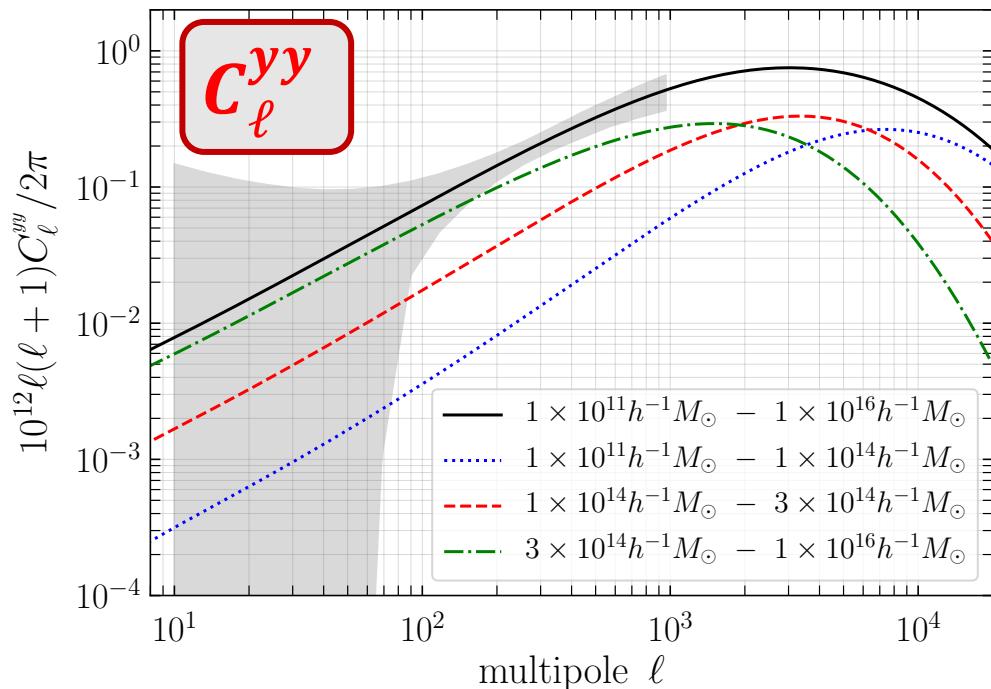


$T_e^{yy}(\ell)$  provides a new map-based observable, complementing the  $y$ -map power spectrum  $C_\ell^{yy}$ , to constrain cosmological parameters

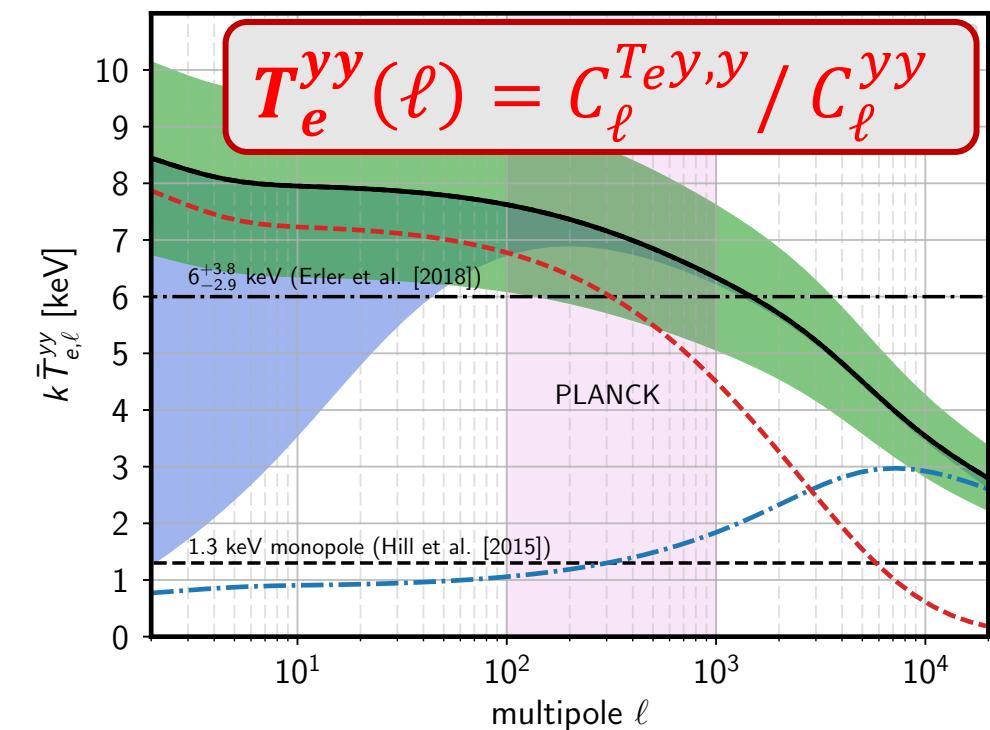
# Cosmology with the relativistic SZ effect

Two independent observables for future cluster cosmology

Classic SZ Compton- $\mathbf{y}$  power spectrum



Electron-temperature  $\mathbf{T}_e$  power spectrum



The shapes of the power spectra  $C_\ell^{yy}$  and  $\mathbf{T}_e^{yy}(\ell)$  have different scaling with cosmological parameters!

# Cosmology with the relativistic SZ effect

Breaking the  $\sigma_8$ - $b$  degeneracy?

$$C_\ell^{yy} = \int_{z_{\min}}^{z_{\max}} dz \frac{dV}{dz d\Omega} \int_{M_{\min}}^{M_{\max}} dM \underbrace{\frac{dn(M, z)}{dM}}_{\substack{\text{halo mass function} \\ \text{different scaling} \\ \text{with } \sigma_8}} \underbrace{|y_\ell(M, z)|^2}_{\substack{\text{pressure profile} \\ \text{same scaling} \\ \text{with mass bias } b}}$$
$$C_\ell^{y,yT_e} = \int_{z_{\min}}^{z_{\max}} dz \frac{dV}{dz d\Omega} \int_{M_{\min}}^{M_{\max}} dM \underbrace{\frac{dn(M, z)}{dM}}_{\substack{\text{temperature-modulated} \\ \text{halo mass function}}} \underbrace{T_e(M)}_{\substack{\text{pressure profile}}} \underbrace{|y_\ell(M, z)|^2}_{\substack{\text{pressure profile}}}$$

$$\Rightarrow T_e^{yy}(\ell) \equiv \frac{C_\ell^{y,yT_e}}{C_\ell^{yy}}$$

depends only on  $\sigma_8$

# Two independent map-based observables for future cluster cosmology

- ✓ The shapes of the power spectra  $\mathbf{T}_e^{yy}(\ell)$  and  $\mathbf{C}_\ell^{yy}$  have different scaling with cosmological parameters
  - $\mathbf{C}_\ell^{yy}$  depends on  $\sigma_8$  and mass-bias  $b$  in a degenerate form, while  $\mathbf{T}_e^{yy}(\ell)$  depends on  $\sigma_8$  but is insensitive to  $b$
- ✓  $\mathbf{T}_e^{yy}(\ell)$  will allow to break parameter degeneracies, possibly alleviating some of the current tensions on cosmological parameters

# Conclusions

- ❖ New component separation approach to disentangle the  $y$  and  $T_e$  observables of the rSZ effect
- ❖ High frequencies  $\gtrsim 300$  GHz are essential to break the  $y-T_e$  degeneracy in rSZ measurements  
*PICO, LiteBIRD, “ESA Voyage 2050” missions would be of great value!*
- ❖ A PICO-type mission would allow us to map rSZ temperatures of thousands of clusters across the entire sky, thus offering a new proxy for determining cluster masses
- ❖ A PICO-type mission would allow us to reconstruct the temperature profiles of many individual clusters, thus offering a deep understanding of the thermodynamics of clusters
- ❖ We may anticipate a “second SZ revolution” in the next decade:
  - ✓ Release of a “ $T_e$ -map” along with the  $y$ -map
  - ✓ Cluster spectroscopy across temperatures
  - ✓ The relativistic electron-temperature power spectrum  $T_e^{yy}(\ell)$  will offer a new map-based observable, complementing  $C_\ell^{yy}$ , to constrain cosmological parameters with clusters

*Thank you!*

*Backup*

# Constrained moment ILC for rSZ

We actually impose additional constraints in the Constrained ILC in order to deproject the kSZ (CMB) contamination and also remove bulk of the dust contamination:

$$\widehat{y\Delta T_e}(\vec{n}) = \sum_{\nu} w(\nu) d(\nu, \vec{n}) \text{ such that } \left\{ \begin{array}{ll} \left\langle (\widehat{y\Delta T_e})^2 \right\rangle \text{of minimum variance} & \\ \sum_{\nu} w(\nu) \partial_{T_e} f^{\text{rSZ}}(\nu, \bar{T}_e) = \mathbf{1} & \text{rSZ: 1}^{\text{st}}\text{-order moment} \\ \sum_{\nu} w(\nu) f^{\text{rSZ}}(\nu, \bar{T}_e) = \mathbf{0} & \text{rSZ: 0}^{\text{th}}\text{-order moment} \\ \sum_{\nu} w(\nu) f^{\text{CMB+kSZ}}(\nu) = \mathbf{0} & \text{kSZ, CMB} \\ \sum_{\nu} w(\nu) f^{\text{dust}}(\nu, \bar{\beta}_d, \bar{T}_d) = \mathbf{0} & \text{dust: 0}^{\text{th}}\text{-order moment} \end{array} \right.$$

$$w = e^T (A^T C^{-1} A)^{-1} C^{-1} A$$

$$A = [\partial_{T_e} f^{\text{rSZ}} \ f^{\text{rSZ}} \ f^{\text{CMB+kSZ}} \ f^{\text{dust}}] \quad e = [1 \ 0 \ 0 \ 0 \ 0]^T$$